Classification of Differential Equations

- Partial Differential Equations $\frac{\mathrm{d}f}{\mathrm{d}x_1}(\vec{x}) = g_0(\vec{x}) + \sum_{j=2}^{\dim} g_j(\vec{x})\frac{\mathrm{d}x_j}{\mathrm{d}x_1}$

  Not in AM II, but terms like Gradient, Jacobian, Hessian, directional derivative, tangential plane, theorem of Schwarz are needed

- Ordinary Differential Equations (ODE): $F(x, y, y', \cdots, y^{(n)}) = 0$
  - Explicit, homogen, first order ODE: $y'(x) - f(x, y(x)) = 0$
    * Calculation of the antiderivative: $y'(x) = f(x)$ Integration
      · Partial Integration (p.I.)
      · Substitution Method (Sub.)
      · Partial Fraction Decomposition (PFD)
      · Trapeziodial Rule (num.)
      · Simpson's Rule (num.)
    * Seperabel ODE: $y'(x) = f(y(x))g(x)$ Separation of Variables
    * Similarity ODE: $y'(x) = f(\frac{y(x)}{x})$ Sub. $z(x) = \frac{y(x)}{x}$
    * Bernouli ODE: $y'(x) + g(x)y(x) + h(x)(y(x))^{\alpha}$ Sub. $z(x) = y(x)^{1-\alpha}$
  - Explicit, homogen, n-th order ODE ($n > 1$): $y^{(n)}(x) - f(x, y, y', \cdots, y^{(n-1)}) = 0$
    * ODE with Constants Coefficients: $\sum_j a_j y^{(j)}(x) = 0$ Ansatz: $y(x) = \mathrm{e}^{\lambda x}$
    * Euler ODE: $\sum_j a_j x^j y^{(j)}(x) = 0$ Sub. $y(x) = z(\ln|x|)$ or Power Ansatz: $y(x) = x^{\lambda}$
    * Autonomous ODE: $y^{(n)}(x) = f(y, y', \cdot, y^{(n-1)})$ Sub. $p(y) = y'(x(y))$
    * Bessel ODE: $x^2 y''(x) + xy'(x) + (x^2 - n^2)\, y(x) = 0$ No closed form - definition of the Bessel function $J_n(x)$
    * Reduction of Order (of d'Alembert) $y(x) = y_0(x)z(x)$ where $y_0(x)$ is a solution of the ODE
    * Euler Method (num.)
    * Runge-Kutta Method (num.)
  - Explicit, non homogen ODE: $y^{(n)}(x) - f(x, y, y', \cdots, y^{(n-1)}) = g(x)$
    $y(x) = y_{hom}(x) + y_p(x)$
    * Variation of Constants $y_{c_1,c_2,\cdot,c_n}(x) \longrightarrow y_{c_1(x),c_2(x),\cdot,c_n(x)}(x)$
    * Undetermined Coefficients
  - Linear Systems of ODEs:
    * First order: $\vec{y}(\vec{x}) = \mathcal{A}\vec{x}$ Calculation of Eigenvalues and Eigenvectors
      $\vec{y}(x) = \sum_{\lambda} c_{\lambda} \mathrm{e}^{\lambda x} \vec{e}_{\lambda}$

  - Power Series Method $y(x) = \sum_{j=0}^{\infty} a_j x^j$
  - Laplace Transformation $(\mathcal{L}y)(s) = \int_0^{\infty} \mathrm{e}^{-sx} y(x) dx$
  - Fourier Transformation $(\mathcal{F}y)(s) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{e}^{-isx} y(x) dx$

If initial values for the function (and if necessary for their derivations) are given one call it an Initial Value Problem (IVP)