Further exercises

Give the solutions of the following ODEs and IVPs:

$$y''(x) = 17(y'(x))^2 \sin(3y(x)), \qquad y(2) = 5,\ y'(2) = 4 \tag{93}$$

Note : The last integral need not to be calculated.

$$y'(x) = \frac{xy(x) + (y(x))^2}{x^2}, \qquad y(\mathrm{e}) = 1 \tag{94}$$

$$\vec{y}'(x) = \begin{pmatrix} 1 & 5 \\ -1 & -1 \end{pmatrix} \vec{y}(x), \qquad \vec{y}(0) = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{95}$$

$$y'(x) = 2x \cos(x^2)\mathrm{e}^{-y(x)}, \qquad y(0) = 1 \tag{96}$$

$$\vec{y}'(x) = \frac{1}{2}\begin{pmatrix} 7 & -1 \\ 1 & 9 \end{pmatrix} \vec{y}(x) \tag{97}$$

$$y'(x) = x^4 y(x) + x^4 (y(x))^4 \tag{98}$$

$$y'(x) = \frac{y(x)}{x} + x^2, \qquad y(1) = 1 \tag{99}$$

$$y'(x) = -\frac{1}{1+x^2}(y(x))^3, \qquad y(0) = 1 \tag{100}$$

$$y'(x) = \frac{x}{1+x^2} y(x) + (1+x^2)^{\frac{3}{2}}(1-x^2), \qquad y(0) = 2 \tag{101}$$

$$y'(x) = \frac{1+x}{3y(x)} + y(x), \qquad y(0) = 1 \tag{102}$$

$$\vec{y}'(x) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \vec{y}(x), \qquad \vec{y}(0) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{103}$$

$$y'(x) = \mathrm{e}^{-\frac{4}{3}x^3}(y(x))^5 + (x^2 - 2)y(x), \qquad y(0) = 2 \tag{104}$$

Determine the solutions of the following ODEs and IVPs:

$$y'''(x) - 3y'(x) - 2y(x) = 6\mathrm{e}^{-x} \tag{105}$$

$$y'(x) + y(x)\tanh x + \frac{1}{2}(y(x))^3(\cosh x)^2 = 0, \qquad y(0) = 1 \tag{106}$$

$$y^{(4)}(x) - 2y'''(x) + 5y''(x) - 8y''(x) + 4y(x) = 0, \qquad y(0) = y''(0) = 0 \tag{107}$$

$$y'''(x) - 9y''(x) + 15y'(x) - 7y(x) = \mathrm{e}^{-x} \tag{108}$$

$$y^{(4)}(x) - 2y''(x) + y(x) = \sin x - \cos x \tag{109}$$

$$y'''(x) - 4y''(x) + 4y'(x) = 0 \tag{110}$$

$$y'''(x) - y''(x) - y'(x) + y(x) = \sin x + \cos x \tag{111}$$

$$y'''(x) + y''(x) - 5y'(x) + 3y(x) = 3x^2 - 4x - 5 \tag{112}$$

$$y^{(4)}(x) - y'''(x) - y''(x) - 2y(x) = -2x^2 - 3x \tag{113}$$

Calculate the following integrals:

$$\int \frac{x}{\sqrt{x+1}(4x+5)}\mathrm{d}x \qquad \int \frac{x^2+x-1}{x^3+x}\mathrm{d}x \tag{114}$$

$$\int x^5\mathrm{e}^{x^3}\mathrm{d}x \qquad \int \sqrt{x}\mathrm{e}^{\sqrt{x}}\mathrm{d}x \tag{115}$$

$$\int (\ln x)^2\mathrm{d}x \qquad \int \sin(\ln x)\mathrm{d}x \tag{116}$$

$$\int \sin x \arctan(\sin x)\mathrm{d}x \qquad \int \frac{1}{1+2\tan^2 x}\mathrm{d}x \tag{117}$$

$$\int_{-1}^{2} \frac{\ln x}{x^2}\mathrm{d}x \qquad \int_0^1 \frac{x+1}{x^2+1}\mathrm{d}x \tag{118}$$