1 Mathematics

- Functions: $f : A \to B, x \mapsto f(x)$
- Derivatives: $f'(x) = \frac{\mathrm{d}f}{\mathrm{d}x}$, in case of time t: $f'(t) = \dot{f}(t)$
  One needs to know derivatives of $const$, x^n, e^x, log, sin, cos
  as well as product, chain, and quotient rules (s. AM I & II).
- Antiderivatives: $F(x) = \int f(x)\,\mathrm{d}x$ and the definite integrals
  One needs to know the product and substitution rules (s. AM I & II).
- Vectors $\vec{x}$ including addition $\vec{x} + \vec{y}$, absolute value $|\vec{x}|$, scalar multiplication $\alpha\vec{x}$, dot product $\vec{x} \cdot \vec{y}$ and cross product $\vec{x} \times \vec{x}$
  as well as different kind of coordinate systems like cartesian, polar and spherical.
- Difference to physics: Don't forget the units!

2 Kinematics

- constant average velocity
  $\bar{v}(t) = \frac{x(t)}{t} = const$
- zero acceleration $a(t) = \dot{v}(t) = 0$

$$v(t) = v_0$$

$$x(t) = \int_0^t v_0\,dt_2 + x_0$$

$$= v_0 t + x_0$$

varying average velocity $\bar{v}(t) \neq const$
One needs actual velocity $v(t) = \dot{x}(t)$
non-zero acceleration $a(t) \neq 0$

$$v(t) = \int_0^t a(t_1)\,dt_1 + v_0$$

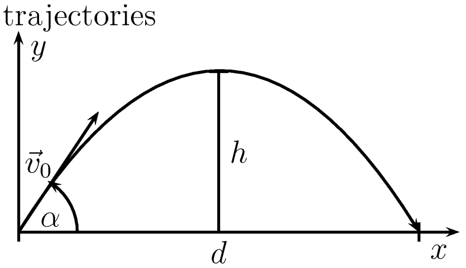
$$x(t) = \int_0^t \int_0^{t_2} a(t_1)\,dt_1 dt_2 + \int_0^t v_0 dt_2 + x_0$$

$$= \frac{1}{2}at^2 + v_0 t + x_0, \text{ for constant acceleration}$$

Generall description of $a(t)$ by Newtons second law $F = ma = m\ddot{x}$

Applications

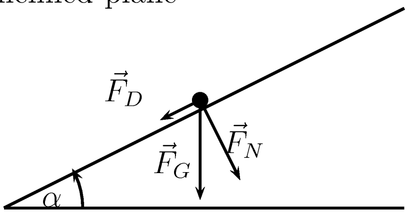
- trajectories



$$0 = F_x = m\ddot{x} \qquad (\dot{x}(0) = v_0 \cos\alpha,\ x(0) = 0)$$

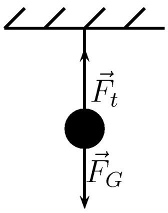
$$-mg = F_y = m\ddot{y} \qquad (\dot{y}(0) = v_0 \sin\alpha,\ y(0) = 0)$$

- inclined plane



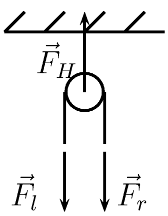
$F_D = F_G \sin\alpha$
$F_N = F_G \cos\alpha$

- robes



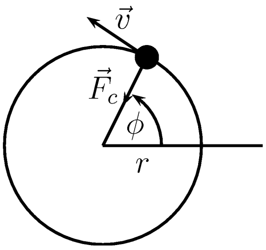
$\vec{F}_t = -\vec{F}_G$

- pulleys



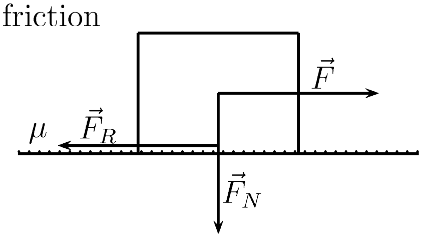
$\vec{F}_H = -(\vec{F}_l + \vec{F}_r)$
$\vec{F} = \vec{F}_l - \vec{F}_r = m\vec{a}$

- orbits



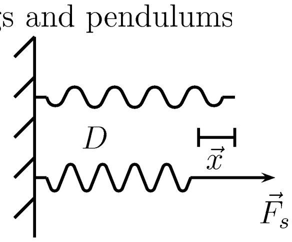
$\omega(t) = \dot{\phi}(t) = \frac{2\pi}{T} = 2\pi f$
$v = |*\vec{v}(t)|* = \omega r$
$a = |*\vec{a}(t)|* = \omega^2 r$
$F_c = \frac{mv^2}{r}$

- friction



static: $\vec{F}_s = \mu\vec{F}_N$ for $\vec{v} = \vec{0}$
dynamic/Stokes: $\vec{F}_R = -\gamma_S\vec{v}$

- springs and pendulums



$\vec{F}_s = -D\vec{x}$

Exercises

1. A stone $m = 1\text{kg}$ is thrown from atower with height $h = 20\text{m}$ horizontally with velocity $v_0 = 10\frac{\text{m}}{\text{s}}$. Calculate the distance d from the tower at which the stone hits the ground.

2.* A skier at position (0,0,0) jumps with velocity $\vec{v}_0 = 10\frac{\text{m}}{\text{s}}\vec{e}_x$ from a hill which can be modelled by $z(x, y) = -x$. Compute the distance d and angle α, the skier hits the ground.

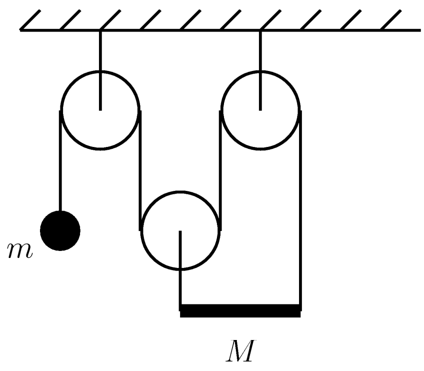
3. A mass $m = 1\text{kg}$ is released at a hill (which can be modelled as an inclined plane with $\alpha_1 = 60°$) at height $h = 1\text{m}$ and slides at the position $h = 0\text{m}$ upwards on another hill ($\alpha_2 = 30°$). Confirm the conservation of energy.

4. A picture with mass $m = 5\text{kg}$ is hung on an robe with maximum tension force $F_{t,max} = 40\text{N}$ at height $h = 10\text{m}$. Is the robe holding, if not when does it hits the ground?

5. For a train with an engine of $F_e = 3\text{kN}$ and three waggons ($m_1 = 100\text{kg}$, $m_2 = 50\text{kg}$, $m_3 = 150\text{kg}$, in order of alginment after the engine), determine the minimum tension force of the robes connecting the waggons (and the engine) and the total acceleration.

6. Two masses are connected by a pulley on two different inclined planes ($\alpha_1 = 30°$, $\alpha_2 = 60°$). For which mass m_2 does $m_1 = 1\text{kg}$ not move?

7.** Calculate the acceleration of $M = 4\text{kg}$ of the following system ($m = 1\text{kg}$):



8. A satellite should be placed in a height of $h = 200\text{km}$ above ground level ($r_{Earth} = 6370\text{km}$). Which horizontal velocity does it needs to get a stabil orbit? What is the time of circulation (in minutes)? (Note: In a height of 200km g is $9.22\frac{\text{m}}{\text{s}^2}$.)

9. A box with mass $m_1 = 3\text{kg}$ lies on a steel table and is connected by a pulley to another free hanging box of $m_2 = 2\text{kg}$. Can the box be built with sheet metal or steel, so that the boxes are not moving? (Note: $\mu_{steel\ on\ steel} = 0.7$, $\mu_{sheet\ metal\ on\ steel} = 0.5$)