3 Energy Conservation

- Different types of energy:
  - mechanical work $E_{mech} = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}(\vec{x})\mathrm{d}\vec{x}$
  - kinetic energy $E_{kin} = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}(\vec{x})\mathrm{d}\vec{x} = m \int_{\vec{x}_1}^{\vec{x}_2} \vec{a}(\vec{x})\mathrm{d}\vec{x} = m \int_{\vec{x}_1}^{\vec{x}_2} \frac{\mathrm{d}\vec{v}}{\mathrm{dt}}\mathrm{d}\vec{x} = \int_{t_1}^{t_2} \frac{\mathrm{d}\vec{v}}{\mathrm{dt}}\vec{v}\mathrm{dt} = m \int_{t_1}^{t_2} \vec{v}\mathrm{d}\vec{v} = \frac{1}{2}m\vec{v}^2$
  - potential energy of gravitation$E_{pot} = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}(\vec{x})\mathrm{d}\vec{x} = -mg \int_{\vec{x}_1}^{\vec{x}_2} \mathrm{d}\vec{x} = -mgh$
    The quantity $V(\vec{x}) = -\int_{\vec{x}_1}^{\vec{x}_2} \vec{F}(\vec{x})\mathrm{d}\vec{x}$ is called potential.
- Conservation of energy if $E(t_1) = E(t_2) = const$ is valid for any times $t_1,\ t_2 \quad \rightarrow \frac{\mathrm{d}E(t)}{\mathrm{d}t} = \lim_{\Delta t} \frac{E(t+\Delta t)-E(t)}{\Delta t} = 0$.
  From Newton's second law and the use of the chain rule we find $\frac{\mathrm{d}E_{kin}}{\mathrm{d}t} = \frac{m}{2}\frac{\mathrm{d}(\dot{x}(t))^2}{\mathrm{d}t} = m\ddot{x}\dot{x} = -V'(x(t))\dot{x} = -\dot{V}(x(t))$. Thus the total Energy $E = E_{kin} + V$ is conserved.

4 Collisions

- Newton third law $\vec{F}_{12} = -\vec{F}_{21}$ leads to $m_1\ddot{\vec{x}}_1 = -m_2\ddot{x}_2 \rightarrow \frac{\mathrm{d}(m_1\dot{x}_1+m_2\dot{x}_2)}{\mathrm{d}t}$. Thus the total momentum $p_{tot} = \sum m_i\dot{x}_i$ is conserved.
- In total elastic collisions the energy is conserved, too.
- In total inelastic collisions the energy is not conserved. Thus the difference $Q = E_{initial} - E_{end}$ is called the heat.

Exercises

1. Consider a mass m on one end of a spring with spring constant D connected to a wall. If the mass is moved x_0 from the equilibrium position what is its velocity when it passes through the equilibrium position?
2. A mass m moves towards an inclined plane with velocity v. At which height does its velocity vanishes?
3. Consider two masses $m_1,\ m_2$ with initial velocities $v_1,\ v_2$ which can only move in one dimension. Calculate the velocities after a central collision.
4. Consider a mass $m_1 = 10\text{kg}$ with initial velocity $\vec{v}_1 = 1\frac{\text{m}}{\text{s}}\vec{e}_x$ which is moving toward a stationary mass $m_2 = 5\text{kg}$. After the collision both masses glues together. What is the velocity $\vec{v}_1'$ after the collision? How much energy is disbated into heat?
5. What is the potential energy of a particle which slides in a salad bowl modeled by $y = cx^2$?
6. Calculate the frequency $f = \frac{1}{2\pi}\omega$ of a particle with mass m sliding in a salad bowl modeled by $y = cx^2$.
7. State the position $x(t)$ and Period $T = \frac{2\pi}{\omega}$ of the oscillation of a mass $m = 1\text{kg}$ attached to spring with constant $D = 4\frac{\text{N}}{\text{m}}$ connected to a wall.
8. Determine the position of the mass of the last system when it slides on a table with stokes constant $\gamma_S = 2\frac{\text{kg}}{\text{s}}$.
9. Consider a uniform rod of mass m and length L which is pivoted at one end. Calculate the moment of inertia and state the frequency $2\pi\omega$ of the oscillation.
10. Determine the moment of inertia of a disk with radius r which is rotated along an axis with distance a to the center of the disk and perpendicular to the disk.