5 Harmonic Oscillator

- Every physical system shich has an equation of motion of the form $m\ddot{x} = -Dx$ or equally has a potential of the form $V(x) = \frac{1}{2}Dx^2$ is called a harmonic oscillator.

- The equation of motion can be solved by the Ansatz $x(t) = A\sin(\omega t + \phi)$, where the phase ϕ and the amplitudes $A,\ B$ are subject to initial conditions. The frequency $2\pi\omega$ is determined by the equation of motion to $\omega = \sqrt{\frac{D}{m}}$

- If in the equation of motion is added a friction term $F_s = -\gamma\dot{x}$ with $\gamma > 0$ the system is called a damped oscillator. The equation can be solved by an exponentially decaying amplitude $A = A_0 \mathrm{e}^{-\gamma t}$

- If another periodic force with frequency $2\pi\Omega$ is added in the equation of motion the oscillator is called an driven harmonic oscillator. The same Ansatz leads to a modified frequency $2\pi\Omega$ and amplitude $A = \frac{A_0}{\omega^2 - \Omega^2}$ with the frequency ω from the ordinary harmonic oscillator. If $\Omega = \omega$ the system is called in the resonant case, where energy is flowing into the system by the external force the whole time.

6 Rigrid Body

- For an rigrid body we seperate the motion of the center of mass $R(t)$ from this of the relative positions $r(t)$ of the particle. For the realive motion the particles can only make a circular motion.

- To get the equation of motion for such motions one takes the cross product of the force or impuls with the relative position r of the particle, leading to the angular momentum $\vec{L} = \vec{x} \times \vec{p}$ and the moment of torque $\vec{M} = \vec{r} \times \vec{F}$. Newtons second law transforms then to $\dot{\vec{L}} = \vec{M}$

- Simplify the relation between the angular momentum and energy of the circulation to the angular velocity ω the moment of interia is indroduced by $\Theta = \sum_i m_i r_i^2$ in the discret version or $\Theta = \int_M r^2(m)\mathrm{d}m = \int_V \rho(r)r^2 \mathrm{d}V$ in the continued case with the density ρ. For another parallel axis at a distance a of the center of mass it can be easily calculated by the parallel-axis theorem $\Theta = \Theta_{cm} + Ma^2$, where Θ_{cm} is the moment of interia of the body if rotated along the parallel axis through the center of mass.

- The kinetic energy of a rigrid body reads $E_{kin} = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\Theta\omega^2$ while the dependence of the angular momentum of the angular velocity is given by $\vec{L} = \Theta\omega\vec{e}_z$.

Exercises

1. Calculate the frequency $f = \frac{1}{2\pi}\omega$ of a particle with mass m sliding in a salad bowl modeled by $y = cx^2$.
2. State the position $x(t)$ and Period $T = \frac{2\pi}{\omega}$ of the oscillation of a mass $m = 1\text{kg}$ attached to spring with constant $D = 4\frac{\text{N}}{\text{m}}$ connected to a wall.
3. Determine the position of the mass of the last system when it slides on a table with stokes constant $\gamma_S = 2\frac{\text{kg}}{\text{s}}$.
4. Consider a uniform rod of mass m and length L which is pivoted at one end. Calculate the moment of inertia and state the frequency $2\pi\omega$ of the oscillation.
5. Determine the moment of inertia of a disk with radius r which is rotated along an axis with distance a to the center of the disk and perpendicular to the disk.