7 Electro- and Magnetostatics

- forces → fields (only dependent on enviroment): $\vec{F} = q\left(\vec{\mathcal{E}} + \vec{v} \times \vec{B}\right)$
- special fields: point charge: $\vec{\mathcal{E}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\vec{e}_r$ homogeneous (condensator): $\vec{\mathcal{E}} = \frac{Q}{A\epsilon_0}\vec{e}_\perp$
  straight wire: $\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{e}_\varphi$ homogeneous (coil): $\vec{B} = \mu_0 I \frac{N}{\ell}\vec{e}_z$
- fields → potential / flux (vector → scalar): $d\mathcal{E} = U = \phi_2 - \phi_1$ $E_{el} = qU$
  $\Phi = BA$ $U_{ind} = -\dot{\Phi}$

Applications

- motion in an electrostatic field (condensator) and magnetostatic field (velocity selector)
- changing the area in a magnetostatic field (generator)

8 Electronics

	connection to potential	connection to geometry	series connection	parallel connection	energy
resistor	$R = \frac{U}{I}$	$R = \rho_s \frac{\ell}{A}$	$R_{eq} = \sum_i R_i$	$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$	$P = UI = \frac{U^2}{R}$
capacitator	$C = \frac{Q}{U}$	$C = \epsilon_r \epsilon_0 \frac{A}{d}$	$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$	$C_{eq} = \sum_i C_i$	$E = \frac{1}{2}CU^2$
inductance	$L = -\frac{U_{ind}}{\dot{I}}$	$L = \mu_r \mu_0 N^2 \frac{A}{\ell}$			$E = \frac{1}{2}LI^2$

Applications

- resitor and capacitator networks,
- antennas (resonant circuits) and stress peaks (opening and closing of circuits)
- electromotors and transformators

Exercises

1. A wire of an electric bulb lies on the potential $\phi_{wire} = 1\text{V}$. From it an electron ($^e/_m = 1,76 \cdot 10^{11}\ ^{\text{C}}/_{\text{kg}}$) is emitted with energy $E_0 = -1\text{eV}$ towards a positive charged plane with potential $\phi_{plane} = 2\text{V}$ at a distance $d = 1\text{m}$. Which velocity has the electron when it hits the plane?

2. A positive point charge $q > 0$ enters a capacitor at the positive plate with velocity v parallel to the plates. The plates of the parallel plate capacitor are separated by a distance d, are of length ℓ and their potential difference amounts to U. Find the velocity, position and angle the point charge leaves the capacitator. For which potential differences does the point charge does not leave the capacitor?

3. Consider a mental inclined plane with angle α, an isolated body with charge q on it and another plane parallel to it at a distance d. Which potential difference is needed to counter the static friction?

4. A metal cube with dimension $\ell \times \ell \times \Delta$ is fully inserted at position x into a capacitator with plates of size $A = \ell^2$ at a distance d which is charged by a voltage U. How much energy is lost in this process?

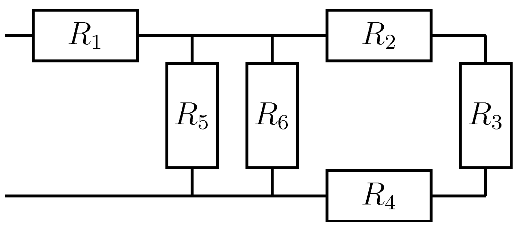
5. A parallel plane capacitor with area $A = \ell^2$ and distance d between the vertical positioned planes is filled with oil to a height h. Determine a formular to calculate the amount of oil by meassuring the charge Q of the capacitor if it is connected to a voltage U.

6. A battery with voltage $U = 10V$ and an inner resistance of $R_b = 5\Omega$ is connected to two serial connected resistors with resistances $R_1 = 35\Omega$ and $R_2 = 60\Omega$. Calculate the current I in the circuit and the voltages U_1 and U_2 that falls at the two resistors.

7. A LED with $R_{LED} = 1\Omega$ is connected to a 1.5V-battery by a serial resistor R. How large have R to be so that the LED will work at a voltage of $U_{LED} = 1\text{V}$? How can this resistance be assembled by 1Ω-resistors?

8. A potentiometer with $R = 120\Omega$ is connected to a voltage $U = 220\text{V}$. Without loading it is fixed to $U_1 = 100\text{V}$. Which voltage U_{load} would be on a load of $R_{load} = 16\Omega$?

9. Calculate the equivalent resistance of the following resistor network with $R_1 = 1\Omega,\ R_2 = 2\Omega,\ R_3 = 3\Omega,\ R_4 = 5\Omega,\ R_5 = 4\Omega,\ R_6 = 6\Omega$:



10. Calculate the equivalent resistance of a cube consisting of resistors of $R = 1\Omega$ at the edges which is connected at two opposite corners.

11. In a mass spectrometer an unknown substance is evaporated by an electric discharge putting the charges $q_i < 0$ on their atoms with energies E_i and masses m_i. Then the particles are accelerated by a potential difference ΔU towards a grid at distance d and passes to a region where a perpendicular electric field $\vec{\mathcal{E}}$ as well as a magnetic field $\vec{B}$ perpendicular to both is applied at a length ℓ. After a pinhole a magnetic field $\vec{B'}$ forces the particles into a circular motion, so that the particles hits the wall of the pinhole at distances x_i from the hole. What voltage ΔU is to apply so every particle has at least the velocity v_{min} when they reaches the gate? In which direction with respect to $\vec{\mathcal{E}}$ has the magnetic field B to point that some particles reaches the hole? Determine the substance if most of the particles hits at $x/\text{cm} = 0.25,\ 0.33,\ 0.4,\ 0.5,\ 0.6,\ 0.67,\ 0.75,\ 0.8,\ \ldots\ 1,\ 1.25,\ 1.33,\ \ldots\ 2,\ 2.25,\ 2.33,\ \ldots$ for $|\vec{\mathcal{E}}| = 1\frac{\text{V}}{\text{m}},\ |\vec{B}| = 2\text{mT},\ |\vec{B'}| = 7\text{mT}$. (Note: $\frac{e}{m_{proton}} = 10^8 \frac{\text{C}}{\text{kg}}$)

12. At $t = 0$ a rectangular frame with length a, width b and resistance R is moving perpendicularly with velocity v into an squared area $A = \ell^2$ where a magnetic field B lies. Neclect effects of self induction and state the timedependend magnetic flux $\Phi(t)$, induced voltage $U_{ind}(t)$ and current $I(t)$ of the frame.

13. A metallic rod with length L is hanging with each end on a rope which are attached at the ceiling at distance ℓ. Find the induced voltage in the rod if there is a magnetic field B perpendicular to the ceiling for small deflections.

14. A metallic rod with length ℓ is moving on two parabolas $y = cx^2$. Calculate the induced voltage in the rod if a magnetic field B is directed along the y-axis and the rod reaches a maximum height h.

15. A metallic rod with resistance R lies on two rails placed at a distance ℓ where a perpendicular magnetic field B into the ground is applied. At $t = 0$ the rails are connected by a condensator C which is charged by a positive charge Q_0 on the right plate. In which direction does the rod moves? Determine the equation of motions and solve them by calculating $Q(t)$.

16. Consider the inside of a metallic rod with dimension $a \times b \times \ell$ to which lies inside a region with magnetic field $B\vec{e}_x$ and which is connected along the z-direciton to a battery with voltage U and internal resistance R. Follow the motion of an electron at the beginning of a current where there are no charges at the side of the rod to the stationary case when a constant current flows in the circuit, thus the electrons are not diverted neither by the magnetic field nor by the surface charges. What is the voltage U_H along the not connected sides of the rod depending on the velocity v of the electrons? Express it in terms of the applied voltage U by considering the current I assembled by n electrons with velocity v passing through a cross section area $A = ab$.