University Karlruhe - International Department

Pre-Semester Physics - Exercises   Summer 2009

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1. **Exercise**

   A mass point with mass $m = 2\,kg$ is placed at a height $h = 1.5\,m$ on a frictionless ramp inclined $30°$ with respect to the horizontal. As the object is released, it slides down the ramp onto a frictionless plane, travels a distance of $d = 30\,cm$, and strikes a spring of spring constant $D = 500\frac{N}{m}$.

   (i) What is the compression of the spring when the mass comes to rest again?

   (ii) How long does it take until the mass meets the spring?

2. **Exercise**

   A car of mass $m_c = 900\,kg$ is racing along a street in a town at a speed $v_c = 20\frac{m}{s}$, well beyond the speed limit. A police car of mass $m_p = 1100\,kg$ standing at the side of the road is spotting racers. As the racer passes by the police car, the police car starts to pursue the racer accelerating at $a = 3\frac{m}{s^2}$. The police car catches up and reaches the racer, unfortunately crashing into it. After the crash, the police car and the racer stick together.

   (i) How long does it take until the police car reaches the racer?

   (ii) What is the speed of the police car then?

   (iii) What is the speed of the cars after the crash?

   (iv) How much energy is lost in the collision?

3. **Exercise**

   A pendulum consists of a mass point with $m = 2\,kg$ attached to a rope of length $l = 3\,m$. The mass point is pushed in horizontal direction with a velocity of $v = 4.5\frac{m}{s}$. Consider the situation at which the rope possesses an angle of $\alpha = 30°$ with respect to the vertical line. How big is

   (i) potential energy?

   (ii) the velocity?

   (iii) Which angle Θ_{max} does it reach at its highest point, again measured with respect to the horizontal line?

4. Exercise

We consider a mass m attached to a spring with spring constant D.

(i) Explain, why the equation of motion takes the form

$$m\ddot{x}(t) + Dx(t) = 0$$

and show that

$$x(t) = A\sin(\omega t) \quad \text{with} \quad \omega = \sqrt{\frac{D}{m}}$$

is a solution.

(ii) Show that the energy E is conserved, i.e.

$$E = \frac{1}{2}m\dot{x}^2(t) + \frac{1}{2}Dx^2(t)$$

5. Exercise

We consider a mass-spring system with a mass $m_1 = 0,4\,kg$, attached to the spring (assumed to be at rest initially) and a second mass $m_2 = 0,6\,kg$ that is moving towards m_1 with velocity v. The two masses collide completely inelastic and start a simple harmonic oscillation with amplitude $A = 16\,cm$ and $f = 0.38\,Hz$.

(i) Find the spring constant D.

(ii) Find the velocity v of mass m_2 before the collision.

(iii) How much energy is lost in the collision?

6. Exercise (more advanced)

In the lecture we saw, that the energy E is a constant of motion

$$E = \frac{m}{2}\dot{x}^2(t) + V(x)$$

This relation shell now serve as a starting point to find $x(t)$. We write

$$\dot{x}(t) = \frac{dx}{dt} = \sqrt{\frac{2}{m}(E - V(x))} \quad \Leftrightarrow \quad \frac{dx}{\sqrt{\frac{2}{m}(E - V(x))}} = dt$$

By integrating both sides of this equation one obtains the function $t(x)$. By taking its inverse one arrives at $x(t)$. Find $x(t)$ for the special case of $V(x) = \frac{D}{2}x^2$.
Hint:

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \arcsin\frac{x}{a}$$