

Pre-Semester Physics - Solutions Summer 2009

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Sheet 1
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1. Exercise

It is $t = t_1 + t_2$. t_1 is the time from the cliff to the sea. Time t_2 the time from the incident to the ground. It is

$$t_1 = \sqrt{\frac{2h_1}{g}} \approx 3.16s.$$

The velocity right at the incidence v_1 is given by

$$v_1 = \sqrt{2gh_1} \approx 31.62 \frac{m}{s}$$

The time t_2 reads

$$t_2 = \frac{h_2}{v_1} = 0.31s.$$

2. Exercise

Choose $x = 0$ and $t = 0$ the position and time where the racer and the police meet. The racer has $x_r(t) = vt$ and the police $x_p(t) = 5t^2$. For the time t we find:

$$vt = \frac{a}{2}t^2 \quad \Leftrightarrow \quad t = \frac{2v}{a} = 4s$$

3. Exercise

It is $t = t_1 + t_2$, where t_1 is the time the stone needs to hit the ground of the fountain and t_2 is the time of the sound needs to reach your ear.

$$t_1 = \sqrt{\frac{2h}{g}}, \quad t_2 = \frac{h}{v_{ac}}$$

Thus

$$t = \sqrt{\frac{2h}{g}} + \frac{h}{v_{ac}} \quad \Leftrightarrow \quad t - \frac{h}{v_{ac}} = \sqrt{\frac{2h}{g}} \quad \Leftrightarrow \quad \left(t - \frac{h}{v_{ac}}\right)^2 = \frac{2h}{g}$$

This yields the required quadratic equation for h :

$$h^2 - 2v_{ac}\left(t + \frac{v_{ac}}{g}\right)h + t^2v_{ac}^2 = 0$$

Putting in the values for $v_{ac} = 340 m/s$, $g = 10 m/s^2$ and $t = 5s$ we obtain $h \approx 109.43m$.

4. Exercise

Let $v = 20 \frac{m}{s}$. With $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$ we have

$$\vec{x}(t) = \begin{pmatrix} \frac{v}{\sqrt{2}}t \\ \frac{v}{\sqrt{2}}t - \frac{g}{2}t^2 \end{pmatrix}, \quad \vec{v}(t) = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ \frac{v}{\sqrt{2}} - gt \end{pmatrix}$$

The total time the mass point is in the air is thus given by

$$\frac{v}{\sqrt{2}}t - \frac{g}{2}t^2 = 0 \quad \Leftrightarrow \quad t = \frac{\sqrt{2}v}{g} \approx 2.82 \text{ s}$$

The horizontal distance d traveled is:

$$d = \frac{v}{\sqrt{2}}t = \frac{v}{\sqrt{2}} \frac{\sqrt{2}v}{g} = \frac{v^2}{g} = 40 \text{ m}$$

The maximum height y_{max} is reached, when the velocity in the y -direction is zero:

$$\frac{v}{\sqrt{2}} - gt = 0 \quad \Leftrightarrow \quad t = \frac{v}{\sqrt{2}g}$$

Thus y_{max} is given by

$$y_{max} = \frac{v}{\sqrt{2}} \frac{v}{\sqrt{2}g} - \frac{g}{2} \left(\frac{v}{\sqrt{2}g} \right)^2 = \frac{v^2}{4g} = 10 \text{ m}$$

5. Exercise

It is

$$v = 45 \frac{km}{h} = 45 \frac{1000 \text{ m}}{3600 \text{ s}} = 12.5 \frac{m}{s}$$

Turning on time around the wheel covers a distance of ($r = 0.35 \text{ m}$)

$$\Delta x = 2\pi r$$

Thus during a leg of $d = 200000 \text{ m}$, the wheels turned n times around with

$$n = \frac{d}{\Delta x} \approx 90991.8$$

For the angular velocity we find

$$\omega = \frac{v}{r} \approx 35.71 \frac{1}{s}$$

The time needed for the leg is

$$\Delta t = \frac{d}{v} = 16000 \text{ s} \approx 4.44 \text{ h}$$

6. Exercise

a) It is

$$\alpha = \arcsin(\omega r / v)$$

b) We have

$$t = r/\sqrt{v^2 - r^2\omega^2}$$

c) Critical angular velocity:

$$\omega = v/r$$

d) We have

$$t = r/v \quad \text{and} \quad \alpha = \omega r/v$$

7. Exercise

It is

$$\dot{\vec{x}}(t) = \begin{pmatrix} e^t(\cos t - \sin t) \\ e^t(\sin t + \cos t) \end{pmatrix}$$

and thus

$$|\dot{\vec{x}}(t)| = \sqrt{2e^{2t}} = \sqrt{2}e^t$$

Therefore the path length L is given by

$$L = \sqrt{2}(e^T - 1)$$