Pre-Semester Physic - Solutions   Summer 2009

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1. **Exercise**

We have

$$a = \frac{F}{m} = \frac{5}{3}\frac{m}{s^2}\,.$$

Thus, the $x(t)$ and $v(t)$ dependencies read

$$x(t) = \frac{5}{6}t^2, \qquad v(t) = \frac{5}{3}t\,.$$

After $3\,s$ the object is moving at a speed

$$v(3s) = 5\frac{m}{s}$$

and it travelled a distance

$$x(3s) = 7.5m\,.$$

2. **Exercise**

The relevant forces acting on the system are $F_1 = -m_1 g \sin\alpha$ and $F_3 = m_3 g$. Thus, with $m_1 = m_3$

$$F = m_1 g(1 - \sin\alpha)$$

The total mass is given by $M = m_1 + m_2 + m_3 = 12m_1$. Thus the acceleration reads

$$a = \frac{F}{M} = \frac{1 - \sin\alpha}{12}g = \frac{g}{24}$$

If we take $x = 0$ and $t = 0$ as the time and the position of m_3 when it is at rest, then we have

$$x(t) = \frac{a}{2}t^2 = \frac{g}{48}t^2$$

We have to solve $x(t) = 0.5\,m$ and thus

$$0.5 = \frac{g}{48}t^2 \quad \Leftrightarrow \quad t = \sqrt{\frac{24}{g}} \approx 1.55\,s$$

We first consider the tension $F_{t,1}$ in the rope at body m_1. It holds:

$$F_z - m_1 g \sin\alpha = m_1 a \quad \Rightarrow \quad F_z = m_1(a + g/2) = m_1 g(1/24 + 1/2) = m_1 g\frac{13}{24}$$

For the tension $F_{t,3}$ in the rope at body $m_3 = m_1$ we obtain

$$m_3 g - F_z = m_3 a \quad \Rightarrow \quad F_z = m_3(g - a) = m_3 g(1 - 1/24) = m_1 g\frac{23}{24}$$

(Note: It holds $m_2 a = F_3 - F_1$.)
If m_2 should stay at rest we must have $|F_1| = |F_3|$:

$$\frac{1}{2} m_1 g = m_3 g \quad \Leftrightarrow \quad \frac{m_1}{m_3} = 2$$

3. **Exercise**

It holds

$$\mu = \tan\alpha \quad \Leftrightarrow \quad \alpha = \arctan\mu = 4.57^\circ$$

The relevant forces for the minimum radius are the centrifugal force $F_c = mv^2/r$, the gravitational force $F_g = mg$ and the static friction $F_s = \mu F_n$. First off all, the normal force F_n consists of two parts. One part stems from the centrifugal force and the other one from the gravitational force

$$F_n = m\frac{v^2}{r}\sin\alpha + mg\cos\alpha$$

and thus

$$F_s = -\mu m(\frac{v^2}{r}\sin\alpha + g\cos\alpha)$$

There is a minus in front, since F_s points in the opposite direction then the centrifugal force does. The other relevant forces along the the inclined plane are the downhill slope force F_d and the part of the centrifugal force pointing along the inclined plane

$$F = -mg\sin\alpha + m\frac{v^2}{r}\cos\alpha$$

To get the minimum radius, the sum of these two must vanish

$$\begin{aligned}
0 &= F_s + F = -\mu m(\frac{v^2}{r}\sin\alpha + g\cos\alpha) - mg\sin\alpha + m\frac{v^2}{r}\cos\alpha \\
0 &= -\mu(\frac{v^2}{r}\sin\alpha + g\cos\alpha) - g\sin\alpha + \frac{v^2}{r}\cos\alpha \\
0 &= \frac{v^2}{r}(\cos\alpha - \mu\sin\alpha) - g(\sin\alpha + \mu\cos\alpha) \\
r &= \frac{v^2}{g}\frac{(\cos\alpha - \mu\sin\alpha)}{(\sin\alpha + \mu\cos\alpha)} = 172.5\,m
\end{aligned}$$

4. **Exercise**

First solve the homogeneous equation

$$m\dot{v} = -\gamma v(t)$$

The solution is

$$v(t) = ce^{-\frac{\gamma}{m}t}$$

For the general solution of

$$m\dot{v} = -\gamma v(t) + mg$$

we make the ansatz $v(t) = c(t)e^{-\frac{\gamma}{m}t}$. This yields:

$$c'(t)e^{-\frac{\gamma}{m}t} = g \quad \Leftrightarrow \quad c'(t) = ge^{\frac{\gamma}{m}t}$$

Thus

$$c(t) = \frac{gm}{\gamma} e^{\frac{\gamma}{m}t} + c$$

and

$$v(t) = (c + \frac{gm}{\gamma} e^{\frac{\gamma}{m}t}) e^{-\frac{\gamma}{m}t} = ce^{-\frac{\gamma}{m}t} + \frac{gm}{\gamma}$$

The boundary conditions $v(0) = 0$ yields $c = -\frac{gm}{\gamma}$. Thus, the solution is finally given by

$$v(t) = \frac{gm}{\gamma}(1 - e^{-\frac{\gamma}{m}t})$$

Note: For $t \to \infty$ we end up with $v = \frac{gm}{\gamma}$. This result may also be obtained by equating the gravitational force and the friction force:

$$mg = \gamma v \quad \Leftrightarrow \quad v = \frac{gm}{\gamma}$$