University Karlruhe - International Department

Pre-Semester Physics - Solutions   Summer 2009

Holger Schmidt
hschmidt@tkm.uni-karlsruhe.de

Sheet 3
28.8.2009

---

1. Exercise

We can use the conservation of energy

$$mgh = \frac{D}{2}s^2 \quad \Leftrightarrow \quad s = \sqrt{\frac{2mgh}{D}} = 0.34\,m$$

When the mass points slides down the inclined plane, it is accelerated with $a = \sin\alpha g = g/2$. The distance it slides down is $l = \frac{h}{\sin\alpha} = 2h$. Thus,

$$x(t) = \frac{a}{2}t^2 = \frac{g}{4}t^2 = 2h \quad \Leftrightarrow \quad t_1 = \sqrt{\frac{8h}{g}} \approx 1.095\,s$$

The velocity when it reaches the horizontal may be otained via energy conservation

$$mgh = \frac{1}{2}mv^2 \quad \Leftrightarrow \quad v = \sqrt{2gh} \approx 5.477\frac{m}{s}$$

Thus, it travels the distance of $d = 0.3\,m$ in

$$t_2 = \frac{d}{v} \approx 0.05\,s$$

Thus, the total time is $t = t_1 + t_2 \approx 1.14\,s$.

2. Exercise

We have $x_p(t) = \frac{a}{2}t^2$ and $x_r(t) = vt$. Thus,

$$x_p(t) = x_r(t) \quad \Leftrightarrow \quad \frac{a}{2}t^2 = v_c t \quad \Leftrightarrow \quad t_1 = \frac{2v_c}{a} = 13.3\,s$$

It is

$$v_p = at_1 = 2v_c = 40\frac{m}{s}$$

The crash is completely inelastic. Thus,

$$m_c v_c + m_p v_p = (m_c + m_p)u \quad \Leftrightarrow \quad u = \frac{m_c v + 2m_p v_c}{m_c + m_p} = 31\frac{m}{s}$$

The energy Q lost in the collision is

$$Q = \frac{m_c}{2}v_c^2 + \frac{m_p}{2}(2v_c)^2 - \frac{m_c + m_p}{2}u^2 = 99kJ$$

3. **Exercise**

The potential energy is given by

$$E_{pot} = mgl(1 - \cos\alpha)$$

For the velocity v' we use

$$\begin{aligned} \frac{1}{2}mv^2 &= \frac{1}{2}mv'^2 + mgl(1 - \cos\alpha) \\ v' &= \sqrt{v^2 - 2gl(1 - \cos\alpha)} = 3,52\frac{m}{s} \end{aligned}$$

For the maximum angle we use

$$\frac{1}{2}mv^2 = mgl(1 - \cos(\Theta_{max})) \quad \Leftrightarrow \quad \Theta_{max} = \arccos(1 - \frac{v^2}{2gl}) = 49°$$

4. **Exercise**

It is

$$\begin{aligned} x(t) &= A\sin(\omega t) \\ \ddot{x}(t) &= -\omega^2 A\sin(\omega t) \end{aligned}$$

If we put this in the eqaution of motion, we obtain

$$(-m\omega^2 + D)\sin(\omega t) = 0$$

Thus,

$$-m\omega^2 + D = 0 \quad \Leftrightarrow \quad \omega = \sqrt{\frac{D}{m}}$$

For the energy conservation we have

$$\begin{aligned} x^2(t) &= A^2\sin^2(\omega t) \\ \dot{x}^2(t) &= \omega^2 A^2\cos^2(\omega t) \end{aligned}$$

Thus,

$$\begin{aligned} \frac{1}{2}m\dot{x}^2(t) + \frac{1}{2}Dx^2(t) &= \frac{1}{2}m\omega^2 A^2\cos^2(\omega t) + \frac{1}{2}DA^2\sin^2(\omega t) \\ &= \frac{1}{2}DA^2(\cos^2(\omega t) + \sin^2(\omega t)) = \frac{1}{2}DA^2 \end{aligned}$$

5. **Exercise**

The total mass is $M = m_1 + m_2 = 1\,kg$. With $\omega = 2\pi f$, the spring constant D is found to be:

$$D = \omega^2 M = 4\pi^2 f^2 M = 5.7\frac{N}{m}$$

The velocity that both masses have right after the collision is

$$u = \omega A = 2\pi f A = 0.38\frac{m}{s}$$

Since mass m_1 is assumed to be initially at rest, it holds:

$$m_2 v = Mu \quad \Leftrightarrow \quad v = \frac{Mu}{m_2} = 0.636\frac{m}{s}$$

The energy lost in the collision is

$$Q = \frac{m_2}{2}v^2 - \frac{M}{2}u^2 = \frac{u^2 M}{2}(\frac{M}{m_2} - \frac{1}{2})$$

6. Exercise

We have

$$\int \frac{dx}{\sqrt{\frac{2}{m}(E - \frac{D}{2}x^2)}} \quad = \quad \int dt$$

If we set $y = \sqrt{\frac{D}{2}}x$ we get

$$\int \sqrt{\frac{m}{D}} \frac{dy}{\sqrt{(E - y^2)}} \quad = \quad \int dt$$

With the hint we obtain

$$\sqrt{\frac{m}{D}} \arcsin \frac{y}{\sqrt{E}} \quad = \quad t$$

or

$$\arcsin \sqrt{\frac{D}{2E}}x \quad = \quad \sqrt{\frac{D}{m}}t$$

Finally, this yield

$$x(t) = \sqrt{\frac{2E}{D}} \sin(\sqrt{\frac{D}{m}}t)$$