University Karlruhe - International Department

**Physics Course - Solutions   Summer 2009**

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**Sheet 4**
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1. **Exercise**

   For $\Theta$ we obtain

$$\Theta = \frac{m}{\pi R^2}\int_0^R 2\pi r r^2 dr = \frac{1}{2}mR^2 = 3125\,kg\,m^2$$

   We have $M = \Theta\dot{\omega}$ and thus

$$\dot{\omega} = \frac{M}{\Theta} \quad \rightarrow \quad \omega(t) = \frac{M}{\Theta}t$$

   It is $\omega(t_1) = \frac{M}{\Theta}t_1 = \omega_1$:

$$M = \frac{\omega_1\Theta}{t_1} = 52,08\frac{kgm^2}{s^2}$$

   For the angle $\phi$ we get

$$\phi(t) = \frac{1}{2}\frac{M}{\Theta}t^2$$

   and thus

$$\phi(t_1) = \frac{1}{2}\frac{M}{\Theta}t_1^2 = 3000$$

   Number of turns

$$N = \frac{3000}{2\pi}$$

   Angular momentum and kinetic energy are given by:

$$L = \Theta\omega_1 = 31250\frac{kg\,m^2}{s} \quad \text{and} \quad E = \frac{1}{2}\Theta\omega_1^2 = 156250\,J$$

2. **Exercise**

   The moment of inertia with respect to the axis of rotation is ($\Theta_{cm} = \frac{1}{12}ML^2$)

$$\Theta = \Theta_{cm} + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

   We have

$$Mg\frac{L}{2} = \Theta\dot{\omega} = \frac{1}{3}ML^2 \quad \Rightarrow \quad \dot{\omega} = \frac{3g}{2L}$$

   Use conservation of energy. The center of mass covered a distance of $L/2$. Thus

$$Mg\frac{L}{2} = \frac{1}{2}\Theta\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{MgL}{\Theta}} = \sqrt{\frac{3g}{L}}$$

**3. Exercise**

We have the following two equations:

$$\begin{aligned} F_T r &= \Theta\dot{\omega} \\ ma &= mg - F_T \end{aligned}$$

Substituting $a = \dot{\omega} r$ in the second equation and solving then both equations for $\dot{\omega}$ we get

$$\dot{\omega} = \frac{mgr}{\Theta + mr^2} = g\frac{1}{r + \frac{\Theta}{mr}}$$

Therefore

$$a = g\frac{1}{r + \frac{\Theta}{mr^2}}$$

This yields for the tension force

$$F_t = \frac{mg}{1 + \frac{mr^2}{\Theta}}$$

**4. Exercise**

See lecture notes.