University Karlsruhe - International Department

Physics Course - Solutions   Summer 2009

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1. Exercise: Competeting forces

Find the ratio of the electric and the gravitational force

(i) between two electrons with mass $m_e = 9.1 \cdot 10^{-31}\,\text{kg}$ which are separated by a distance $r = 1\,\text{m}$.

(ii) for an isotropic ball with mass $m = 1\,\text{kg}$ charged by $q = 1\,\text{C}$ in a thunderstorm generating an homogeneous electric field of $E = 10^6\,\frac{\text{V}}{\text{m}}$ on the earth ($g = 10\,\frac{\text{m}}{\text{s}^2}$).

(iii) between two isotropic planets with masses $M = 6 \cdot 10^{24}\,\text{kg}$ at a distance of $R = 2 \cdot 10^7\,\text{m}$, on each standing one person with mass $m = 80\,\text{kg}$ and a charge excess of 1%. Assume that the neutral persons only consist of neutral carbon atoms with 12 electrons and mass $m_C = 2 \cdot 10^{-26}\,\text{kg}$ and that the persons are separated by a distance of $r = 3\,\text{m}$.

Solution:

(i) The Forces can be calculated by Coulomb's law and its gravitational analogon:

$$\begin{aligned}
F_g &= \gamma \frac{m_e^2}{r^2} = 6.67 \cdot 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{9.1^2 \cdot 10^{-31 \cdot 2}\,\text{kg}^2}{1^2\,\text{m}^2} = 5.5 \cdot 10^{-71}\,\text{N} \\
F_e &= \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{1}{4 \cdot \pi \cdot 8.85 \cdot 10^{-12}\,\frac{\text{C}}{\text{Vm}}} \cdot \frac{1.6^2 \cdot 10^{-12 \cdot 2}\,\text{C}^2}{1^2\,\text{m}^2} = 2.3 \cdot 10^{-28}\,\text{N} \\
\underline{\underline{\frac{F_e}{F_g}}} &= \frac{2.3 \cdot 10^{-28}\,\text{N}}{5.5 \cdot 10^{-71}\,\text{N}} = \underline{\underline{4.2 \cdot 10^{42}}}
\end{aligned}$$

Thus in the microscopic world the electromagnetic force is usually much stronger than the gravitational one, so that the latter one can be neglected.

(ii) Here the following simplified formulas can be used:

$$\begin{aligned}
F_g &= mg = 1\,\text{kg} \cdot 10\,\frac{\text{m}}{\text{s}^2} = 10\,\text{N} \\
F_e &= qE = 1\,\text{C} \cdot 10^6\,\frac{\text{V}}{\text{m}} = 10^6\,\text{N} \\
\underline{\underline{\frac{F_e}{F_g}}} &= \frac{10^6\,\text{N}}{10^1\,\text{N}} = \underline{\underline{10^5}}
\end{aligned}$$

(iii) Since the mass m consists of masses m_C with charge $12e$ it is are charged by $Q = 1\% \cdot 12e \cdot \frac{m}{m_C} = 0.01 \cdot 12 \cdot 1.6 \cdot 10^{-19}\,\mathrm{C} \cdot \frac{80\,\mathrm{kg}}{2\cdot 10^{-26}\,\mathrm{kg}} = 7.7 \cdot 10^{7}\,\mathrm{C}$:

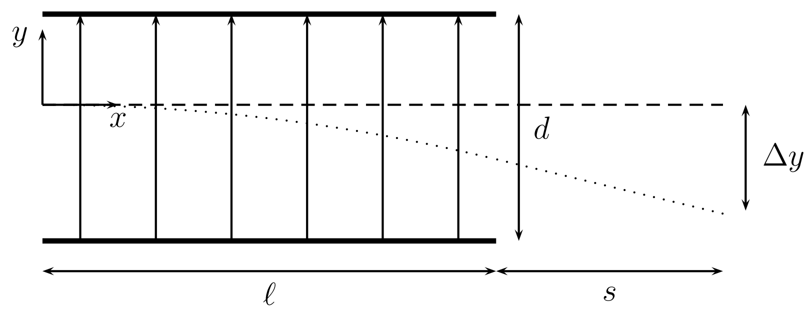
$$\begin{aligned}
F_g &= \gamma \frac{M^2}{R^2} = 6.67 \cdot 10^{-11}\,\frac{\mathrm{Nm}^2}{\mathrm{kg}^2} \cdot \frac{6^2 \cdot 10^{24\cdot 2}\,\mathrm{kg}^2}{2^2 \cdot 10^{7\cdot 2}\,\mathrm{m}^2} = 6.0 \cdot 10^{24}\,\mathrm{N} \\
F_e &= \frac{1}{4\pi\epsilon_0}\frac{Q^2}{r^2} = \frac{1}{4\cdot\pi\cdot 8.85\cdot 10^{-12}\,\frac{\mathrm{C}}{\mathrm{Vm}}} \cdot \frac{7.7^2 \cdot 10^{7\cdot 2}\,\mathrm{C}^2}{3^2\,\mathrm{m}^2} = 5.9 \cdot 10^{24}\,\mathrm{N} \\
\underline{\underline{\frac{F_e}{F_g}}} &= \frac{5.9 \cdot 10^{24}\,\mathrm{N}}{6.0 \cdot 10^{24}\,\mathrm{N}} = \underline{\underline{0.98}}
\end{aligned}$$

Since the persons are strongly charged, the gravitational force becomes dominant at those astronomic scales due to the compensation of the two different electric charges.

2. Exercise: An airplane in a thunderstorm

Background: When airplanes are flying through clouds, they often get electrostaticly charged. In this way they experience an electrostatic force when they try to land in a thunderstorm. They can cause serious problems for radio, too. For compensation, they carry static dischargers, small antenna like objects, at the back of their wings. Additionally they are grounded when they reach the terminals to prevent injuries after landing.

We model the thunderstorm by a square parallel plate capacitor, and the airplane as a point charge where the gravitational force is compensated by the buoyant force. Thus consider a negative point charge with charge $q = -10\,\mu\mathrm{C}$ and mass $m = 500\,\mathrm{kg}$ which moves horizontally with a velocity of $v_0 = 300\,\frac{\mathrm{m}}{\mathrm{s}}$ to the right along the axis of the parallel plate capacitor as shown. The plates of the parallel plate capacitor are separated by a distance $d = 2\,\mathrm{km}$ and have a length of $\ell = 30\,\mathrm{km}$. In the region between the plates an electric field $E = 1 \cdot 10^5\,\frac{\mathrm{V}}{\mathrm{m}}$ is measured which is pointing upwards.



First take a look at the point charge:

(i) What is the absolute value of the electrostatic force on the point charge and in which direction is it pointing?

(ii) How long does the point charge need to pass the parallel plate capacitor?

(iii) What is the distance Δy from the original height where the charge will be positioned after the length $s = 20\,\mathrm{km}$ succeeding the end of the capacitor?

Next consider the parallel-plate capacitor:

(iv) What is the capacitance C?

(v) Calculate the potential difference U.

(vi) Determine the amount of charges Q on the plates.

Solution:

(i) The electrostatic force has an absolute value of

$$\underline{\underline{F_e}} = |q|E = \underline{\underline{1\,\mathrm{N}}}$$

and is pointing $\underline{\underline{\text{downwards}}}$.

(ii) Since the motion is one with constant acceleration

$$\vec{r}(t) = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_0 t \\ -\frac{1}{2}\frac{F_e}{m}t^2 \end{pmatrix}, \qquad \vec{v}(t) = \dot{\vec{r}}(t) = \begin{pmatrix} v_0 \\ -\frac{F_e}{m}t \end{pmatrix}$$

the charge passes the capacitor after the time

$$\underline{\underline{t_0}} = \frac{\ell}{v_0} = \underline{\underline{100\,\mathrm{s}}}\,.$$

(iii) During its time in the capacitor the charge will be deflected by

$$\Delta y_{\mathrm{cap}} = \frac{1}{2}\frac{F_e}{m}t_0^2$$

(motion with constant acceleration). Afterwards it performs an uniform motion with velocities $v_x = v_0$ and $v_y = -\frac{F_e t_0}{m}$ during the time $t_s = \frac{s}{v_0}$. Therefore, after the distance s from the capacitor it will be deflected from its original height by

$$\underline{\underline{\Delta y}} = \Delta y_{\mathrm{cap}} + |v_y| t_s = \frac{1}{2}\frac{F_e}{m}t_0^2 + \frac{F_e t_0}{m}\frac{s}{v_0} = \underline{\underline{23\,\mathrm{m}}}\,.$$

(iv) The capacitance is given by

$$\underline{\underline{C}} = \epsilon_0 \frac{\ell^2}{d} = \underline{\underline{4.0\,\mu\mathrm{F}}}\,.$$

(v) The voltage is determined by the electric field to

$$\underline{\underline{U}} = Ed = \underline{\underline{2 \cdot 10^8\,\mathrm{V}}}\,.$$

(vi) The charges on the plates can therefore be calculated by the definition of the capacitance:

$$C = \frac{Q}{U} \qquad \Rightarrow \qquad \underline{\underline{Q}} = CU = \underline{\underline{797\,\mathrm{C}}}\,.$$

3. Exercise: The coaxial cable

Background: Coaxial cables are the simplest cables which screen their transmitted information from the environment. The characteristics of high frequency communication of these cables are strongly influenced by their capacitance.

Consider two metallic cylinders with the same axis, height ℓ, radius a and $b > a$. They are charged by $Q > 0\,\mathrm{C}$ and $-Q$. Neglect the effects of both bases.

(i) From Gauss' law, what is the electric field inside the small cylinder ($r < a$)?

(ii) What is the electric field outside the small cylinder ($r > b$)?

(iii) Draw the electric field in a plane perpendicular to the central axis.

(iv) From symmetry, state a surface where the electric field E is constant.

(v) Use the first Maxwell equation to determine the electric field as function of r for $a < r < b$.

(vi) Find the voltage U.

(vii) Calculate the capacitance C for a system with $a = 1\,\mathrm{mm}$, $b = 5\,\mathrm{mm}$ and $\ell = 1\,\mathrm{m}$.

Solution:

(i) The electric field vanishes

$$\underline{\underline{E_< = 0\,\frac{\mathrm{V}}{\mathrm{m}}}}$$

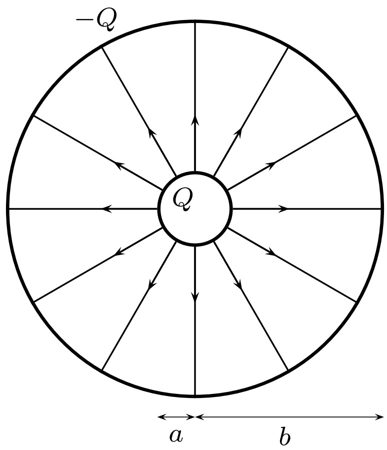
for $r < a$ since no charges are inside a Gauss' surface.

(ii) The electric field is negligible

$$\underline{\underline{E_> = 0\,\frac{\mathrm{V}}{\mathrm{m}}}}$$

for $r > b$ as well, since the charges inside a Gauss' surface adding up to $0\,\mathrm{C}$.

(iii) Due to the symmetry, the electric field goes radial from the positively charged inner cylinder to the negatively charge outer cylinder for $a < r < b$:



(iv) Along a cylinder barrel around the same axis the electric field should be constant. (The contributions of the bases are neglected!)

(v) The radial electric field can be determined by the Gauss' surface to

$$\int_{\text{cylinder barrel}} E_r \mathrm{d}A = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad E_r \cdot 2\pi r \ell = \frac{Q}{\epsilon_0} \quad \Rightarrow \quad \underline{\underline{E_r = \frac{Q}{2\pi\epsilon_0 \ell r}}} \,.$$

(vi) When the electric field is integrated the voltage can be deduced:

$$\underline{\underline{U}} = \int_a^b E_r \mathrm{d}r = \frac{Q}{2\pi\epsilon_0\ell} \int_a^b \frac{1}{r} \mathrm{d}r = \underline{\underline{\frac{Q}{2\pi\epsilon_0\ell} \ln\frac{b}{a}}}$$

(vii) From this quantity the capacitance is calculated to

$$\underline{\underline{C}} = \frac{Q}{U} = \frac{2\pi\epsilon_0\ell}{\ln\frac{b}{a}} = \underline{\underline{34.6\,\mathrm{pF}}} \,.$$

Note:

- Permitivity of free space: $\epsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\mathrm{C}}{\mathrm{Vm}}$
- Gravitation constant: $\gamma = 6.67 \cdot 10^{-11}\,\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$
- Elemetary charge: $e = 1.6 \cdot 10^{-19}\,\mathrm{C}$
- The antiderivative $\int \frac{1}{r}\mathrm{d}r = \ln r$