Physics Course - Solutions   Summer 2009

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Sheet 7
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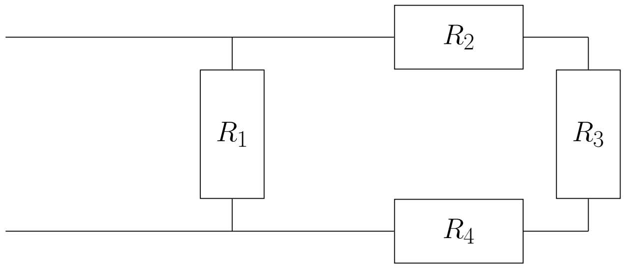
1. Exercise: Resistor network

Consider the following the following resistor network with $R_1 = 2\,\Omega$, $R_2 = R_4 = 1\,\Omega$:



(i) Determine the resistance R_3 when the resistor consists of a $\ell = 4\,\mathrm{m}$ long Nichrome wire ($\rho_s = 10^{-6}\,\Omega\,\mathrm{m}$) with radius of $r = 0.65\,\mathrm{mm}$.

(ii) What is the equivalent resistance between the left points?

(iii) Calculate the current that would flow through the whole system (in the left wires) when between the left points a potential difference of $U = 1\,\mathrm{V}$ is applied.

Solution:

(i) The Nichrome resistor possesses a resistance of

$$\underline{\underline{R_3}} = \rho_s \frac{\ell}{\pi r^2} = \underline{\underline{3.0\,\Omega}}\,.$$

(ii) The series connection of the three resistors to the right gives a resistance of

$$R_\mathrm{r} = R_2 + R_3 + R_4 = 5.0\,\Omega.$$

This resistance is parallel connected to R_1 which together gives a total equivalent resistance of

$$\frac{1}{R_\mathrm{eq}} = \frac{1}{R_\mathrm{r}} + \frac{1}{R_1} = 0.70\,\Omega^{-1} \Rightarrow \underline{\underline{R_\mathrm{eq} = 1.4\,\Omega}}\,.$$

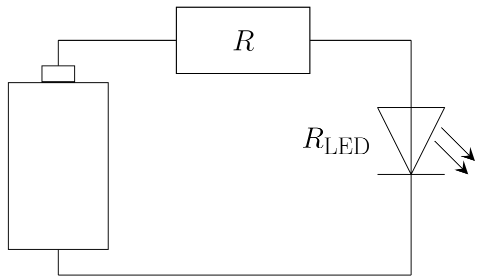
(iii) The current I can be calculated by Ohm's law:

$$R_\mathrm{eq} = \frac{U}{I} \Rightarrow \underline{\underline{I}} = \frac{U}{R_\mathrm{eq}} = \underline{\underline{700\,\mathrm{mA}}}$$

2. **Exercise: Light emitting diodes**

Background: Light emitting diodes (LED) are a fast evolving part of lighting economy. However, diodes possess a very steep increase in their characteristic curve (U-I-diagram) above a critical applied voltage. In this way a naive installation might result in overheating and destruction of the LED. To prevention this, they are often used with series resistors.

Consider a circuit with a LED with inner resistance $R_{\text{LED}} = 100\,\Omega$ (at a voltage of $U_{\text{LED}} = 2\,\text{V}$) and a resistor with resistance R connected to a battery with $U_0 = 3\,\text{V}$ as sketched in the figure below.



(i) Determine the necessary current and from it the necessary series resistance R, if the LED should be operated at $U_{\text{LED}} = 2\,\text{V}$.

(ii) How might this resistance be assembled by 100 Ω-resistors?

(iii) Calculate the power P_{LED} dissipated in the LED.

(iv) Now include an inner resistance of the battery of $R_b = 1\,\Omega$ in your considerations. How large should now be the series resistance?

(v) For this case, state the power P provided by the battery as well as the one dissipated in the series resistor.

Solution:

(i) The necessary current through the circuit is

$$R_{\text{LED}} = \frac{U_{\text{LED}}}{I} \qquad \Rightarrow \qquad \underline{\underline{I}} = \frac{U_{\text{LED}}}{R_{\text{LED}}} = \underline{\underline{20\,\text{mA}}}\,.$$

From Kirchhoff's voltage law it can be deduced that at the series resistor should be a voltage drop of

$$U_R = U_b - U_{\text{LED}} = 1\,\text{V}.$$

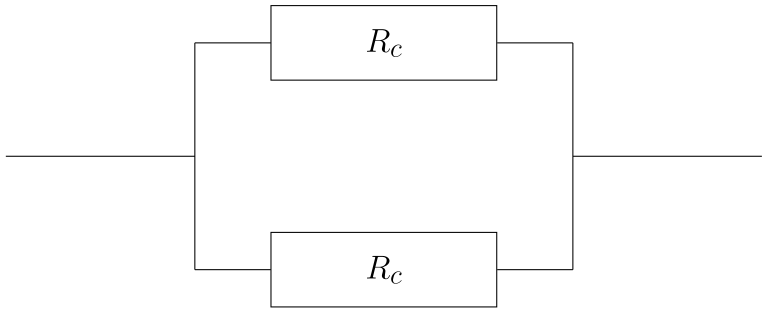
Thus the series resistor should have a resistance of

$$\underline{\underline{R}} = \frac{U_R}{I} = \underline{\underline{50\,\Omega}}\,.$$

(ii) It can be constructed by $\underline{\underline{\text{two parallel } R_c = 100\,\Omega\text{-resistors}}}$, due to

$$R = \frac{1}{\frac{1}{R_c} + \frac{1}{R_c}} = \frac{1}{2} R_c = 50\,\Omega$$

holds:



(iii) The power dissipated in the LED reads

$$\underline{\underline{P_{\text{LED}}}} = U_{\text{LED}} I = \underline{\underline{40\,\text{mW}}}\,.$$

(iv) Since the new series resistor R' and the inner resistance R_b form a series connection which has to have a resistance of $R = 50\,\Omega$ as before, the series resistor should now have a resistance of

$$R = R' + R_b \qquad \Rightarrow \qquad \underline{\underline{R'}} = R - R_b = \underline{\underline{49\,\Omega}}$$

(v) The battery consists of the voltage source and the inner resistance. Therefore between its contacts a potential difference of

$$\Delta U = U_0 - U_b$$

will drop. This difference combined with the current determines the power the battery provides:

$$\underline{\underline{P}} = \Delta U I = (U_0 - U_b) I = U_0 I - R_b I^2 = \underline{\underline{59.6\,\text{mW}}}$$

Here the power dissipated by the inner resistance was determined by Ohm's law. The same formula can be used to calculate the power

$$\underline{\underline{P_R}} = R I^2 = \underline{\underline{19.6\,\text{mW}}}$$

dissipated at the series resistance.

3. Exercise: Conductive level sensor

Background: Conductive level sensors provide an easy way to measure continuously the presence of solids and liquids, e.g. the tank filling height. They are often seen as small cylinders which are inserted in the storage area.

A parallel plane capacitor with area $A = \ell^2$ and distance d between the vertical positioned planes is filled with petrol (relative permittivity ϵ_r) to a height h.

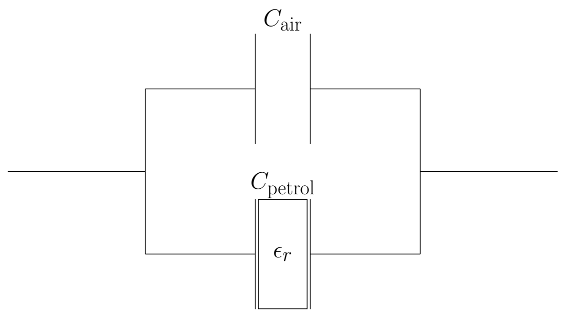
(i) State the equivalent circuit diagram with homogeneous filled capacitors.

(ii) What is the total capacitance of this circuit dependend on its consisting capacitances?

(iii) Determine the dependence of the capacitance of this system from the volume V_{petrol} of petrol in the capacitor?

Solution:

(i) The equivalent circuit consists of $\underline{\underline{\text{two parallel connected capacitors}}}$, one empty and one filled with petrol:



(ii) Since it is a parallel connection the capacitance add up to

$$\underline{\underline{C_{\text{tot}} = C_{\text{air}} + C_{\text{petrol}}}}$$

(iii) With the formulas describing the dependence of the modeled capacitors to theirs geometry the total capacitance is given by

$$\underline{\underline{C_{\text{tot}}}} = \frac{\epsilon_0 \ell}{d}(\ell - h) + \frac{\epsilon_0 \ell}{d}\epsilon_r h = \underline{\underline{C_{\text{empty}} + \frac{\epsilon_0(\epsilon_r - 1)}{d^2} V_{\text{petrol}}}}$$

Thus the total capacitance is linear dependent with the volume of petrol.