University Karlsruhe - International Department

Physics Course - Solutions Summer 2009

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Homework (Sheet 8)
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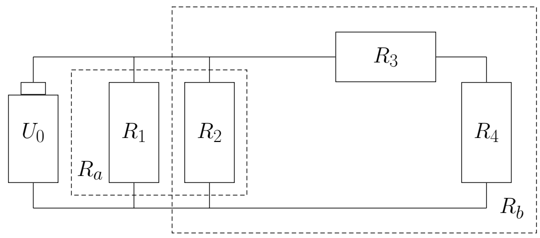
State the ansatz, the formula and the result of the following exercises:

(i) An empty capacitor consists of two parallel planes with area $S = 0.5\,\text{km}^2$ which are separated by a distance of $\ell = 1\,\text{m}$. Calculate the capacity.

(ii) What is the angle φ of a perpendicular triangle if the side opposite to the angle is $\ell = 1\,\text{mm}$ long and the side which is perpendicular to this side has a length of $s = 1\,\text{km}$.

(iii) What is the equivalent resistance R_a and R_b sketched in the following circuit:



Calculate the voltage U_p provided by the battery if it has an internal resistance of $r = \frac{R_1}{10} = \frac{R_2}{10} = \frac{R_3}{10} = \frac{R_4}{10} = 0.1\,\Omega$ and a voltage of $U_0 = 2\,\text{V}$.

(iv) Determine the terminal current I_c through a coil of length $d = 30\,\text{cm}$ with $N_c = 20$ turns and an inner resistance $R_c = 2\,\Omega$, which is connected to a battery of $U_b = 1.5\,\text{V}$ with an internal resistance of $R_b = 1\,\Omega$. How large would be the magnetic field inside the coil?

(v) How is the magnetic field B at a distance $\ell = 20\,\text{cm}$ of a wire connected to the current I_w that is running through it?

(vi) What motion performs an electron with mass m_e and charge e in a magnetic field $\vec{B} = B_z\vec{e}_z$? State the position $\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$ of the electron from its origin at $t = 0\,\text{s}$ where it moves with the velocity v_0 parallel to the x-axis.

(vii) Determine the potential difference $\Delta\phi$ that has to be applied to two parallel plates which are separated by a distance $\ell = 1\,\text{m}$, so that electrons passing through them with velocity $v_e = 10\,\frac{\text{m}}{\text{s}}$ are not deflected, if between them a magnetic field of strength $B = 100\,\text{mT}$ parallel to the plates and perpendicular to the velocity of the electrons is present.

Note:

- Elementary charge: $e = 1.6 \cdot 10^{-19}\,\mathrm{C}$
- Permittivity of free space: $\epsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$
- Permeability of vacuum: $\mu_0 = 1.26 \cdot 10^{-6}\,\frac{\mathrm{Vs}}{\mathrm{Am}}$
- Lorentz force: $\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B})$

Solution:

(i) Here we can use the formula from the lecture as ansatz:

$$\underline{\underline{C}} = \underline{\epsilon_0 \frac{S}{\ell}} = 8.8 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}} \frac{5 \cdot 10^5\,\mathrm{m}^2}{1\,\mathrm{m}} = \underline{\underline{4.4\,\mu\mathrm{F}}}$$

(ii) From the definition of the tangents the angle can be derived:

$$\underline{\tan\varphi = \frac{\ell}{s}} = \frac{10^{-3}\,\mathrm{m}}{10^3\,\mathrm{m}} = 10^{-6} \qquad \Rightarrow \qquad \underline{\underline{\varphi}} = \underline{\arctan\left(\frac{\ell}{s}\right)} = \underline{\underline{5.7 \cdot 10^{-5\,\circ} = 10^{-6}}}$$

(iii) From the formula of the series and parallel resistors the to equivalent resistors can be determined:

$$\underline{\underline{R_a}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \underline{\underline{\frac{R_1 R_2}{R_1 + R_2}}}$$

$$\underline{\underline{R_b}} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + R_4}} = \underline{\underline{\frac{R_2 R_3 + R_2 R_4}{R_2 + R_3 + R_4}}}$$

To calculate the voltage drop at the battery we state the equivalent resistance which lies at the voltage source of the battery

$$\underline{R_{eq} = r + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3 + R_4}}} = 0.1\,\Omega + \frac{1}{1\,\Omega^{-1} + 1\,\Omega^{-1} + 0.5\,\Omega^{-1}} = 0.5\,\Omega,$$

from which we can derive the current $I_b = \frac{U_0}{R_{eq}}$ and from it the voltage drop at the internal resistor by Ohm's law:

$$U_r = r I_b = r \frac{U_0}{R_{eq}}$$

This can now be considered in Kirchhoff's voltage law to find the voltage provided by the battery:

$$\underline{\underline{U_p}} = U_0 - U_r = \underline{U_0 - r\frac{U_0}{R_{eq}}} = 2\,\mathrm{V} - 0.1\,\Omega \frac{2\,\mathrm{V}}{0.5\,\Omega} = \underline{\underline{1.6\,\mathrm{V}}}$$

(iv) Here, we define again the equivalent resistance that lies at the voltage source:

$$\underline{R_{eq} = R_c + R_b} = 2\,\Omega + 1\,\Omega = 3\,\Omega$$

From it and from Ohm's law we can calculate the current through the coil which is by Kirchhoff's current law equivalent to the current through the whole system:

$$\underline{\underline{I_c}} = \frac{U_b}{\underline{R_{eq}}} = \frac{1.5\,\text{V}}{3\,\Omega} = \underline{\underline{500\,\text{mA}}}$$

This current we can then put into the formula derived in the lecture for the magnetic field of a coil:

$$\underline{\underline{B}} = \underline{\mu_0 \frac{N_c}{d}} I_c = 1.26 \cdot 10^{-6}\,\frac{\text{Vs}}{\text{Am}}\frac{2 \cdot 10^1}{3 \cdot 10^{-1}\,\text{m}} \cdot 5 \cdot 10^{-1}\,\text{A} = \underline{\underline{42\,\mu\text{T}}}$$

(v) The magnetic field is given by the formula of the lecture of the magnetic field of a wire:

$$\underline{\underline{B}} = \underline{\frac{\mu_0}{2\pi\ell}} I_w = \frac{1.26 \cdot 10^{-6}\,\frac{\text{Vs}}{\text{Am}}}{2\pi \cdot 2 \cdot 10^{-1}\,\text{m}} I_w = \underline{\underline{10^{-6}\,\frac{\text{T}}{\text{A}} I_w}}$$

(vi) A circular motion has the position-time dependence:

$$\underline{\vec{r}(t) = \begin{pmatrix} r_0 \cos(\omega t + \phi) + r_x \\ r_0 \sin(\omega t + \phi) + r_y \\ 0 \end{pmatrix}}$$

In putting in the starting values ($\vec{r}(0) = \vec{0}$ and $\vec{v}(0) = v_0 \vec{e}_x$) and identify the centripetal force as the magnetic force we obtain

$$\begin{aligned} \underline{\underline{\vec{r}(t)}} &= \underline{\underline{\begin{pmatrix} r_0 \sin \omega t \\ r_0(\cos \omega t - 1) \\ 0 \end{pmatrix}}} \\ \text{with} \qquad \underline{\underline{\omega}} &= \sqrt{\frac{|\vec{a}|}{r_0}} = \sqrt{\frac{|\vec{F}_L|}{m_e r_0}} = \underline{\underline{\sqrt{\frac{e v_0 B_z}{m_e r_0}}}} \\ \text{and} \qquad v_0 &= r_0 \omega = \sqrt{\frac{e v_0 B_z r_0}{m_e}} \qquad \Leftrightarrow \qquad \underline{\underline{r_0 = \frac{m_e v_0}{e B_z r_0}}}\ . \end{aligned}$$

(vii) From the necessary balance of the magnetic and electrostatic force

$$\underline{F_B = F_E} \qquad \Rightarrow \qquad e v_e B = eE = e\frac{\Delta\phi}{\ell}$$

the potential difference can be derived:

$$\underline{\underline{\Delta\phi}} = \underline{v_e B \ell} = 10\,\frac{\text{m}}{\text{s}} \cdot 0.1\,\frac{\text{Vs}}{\text{m}^2} \cdot 1\,\text{m} = \underline{\underline{1\,\text{V}}}$$