University Karlsruhe - International Department

Physics Course - Solutions Summer 2009

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Sheet 9
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1. Exercise: A falling frame

At $t = 0\,\mathrm{s}$ a quadratic metallic frame with length $a = 1\,\mathrm{mm}$ is moving with velocity $v = 1\,\frac{\mathrm{m}}{\mathrm{s}}$ into a coil parallel to its $N = 100$ turns. Through the coil with length $\ell = 12.6\,\mathrm{cm}$ and radius $r = 1\,\mathrm{cm}$ flows a current of $I = 1\,\mathrm{A}$. Consider the radius of the coil is sufficiently large and neglect effects of self induction as well as the magnetic field outside the coil.

(i) Calculate the magnetic field of the coil.

(ii) How would this quantity change when iron ($\mu_r = 1000$) is inserted into the coil? (The relative permeability μ_r describes the relative *increase* of the magnetic field by matter in the same way the relative permittivity ϵ_r does for the relative *decrease* of the electric field.)

(iii) Draw the timedependent magnetic flux $\Phi(t)$ for the empty coil.

(iv) What is the maximal induced voltage $\hat{U}_{\text{ind}}$ for the empty coil?

(v) When can this value be measured?

(vi) In which direction is the current flowing through the frame?

Solution:

(i) Using the formula derived in the lecture, the magnetic field is given by

$$\underline{\underline{B}} = \mu_0 \frac{N}{\ell} I = \underline{\underline{1\,\mathrm{mT}}}$$

(ii) Since for homogeneous field

$$\epsilon_r = \frac{C_r}{C} = \frac{U}{U_r} = \frac{E}{E_r} \Rightarrow E_r = \frac{1}{\epsilon_r} E$$

can be derived (the index r denotes the quantities in presence of a dielectric material) the magnetic field enhances to

$$\underline{\underline{B_r}} = \mu_r B = \underline{\underline{1\,\mathrm{T}}}$$

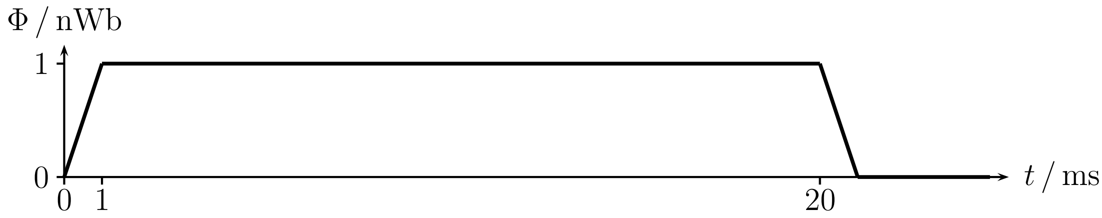
(iii) The magnetic flux for a homogeneous field and an increasing area is given by

$$\Phi(t) = BA = Bavt = 1\,\mu\mathrm{V}t \quad \text{for } 0 < t < \frac{a}{v} = 1\,\mathrm{ms}.$$

After that last time index the flux will be constant $\Phi(t) = 1\,\mathrm{nWb}$ until it will start to decrease due to a decreasing area:

$$\Phi(t) = 1\,\mathrm{ns} - 1\,\mu\mathrm{V}(t - 20\,\mathrm{ms}) \quad \text{for } \frac{2r}{v} = 20\,\mathrm{ms} < t < \frac{2r}{v} + \frac{a}{v} = 21\,\mathrm{ms}$$

Otherwise it will vanish.



(iv) Since the orientated voltage is

$$U_{\text{ind}} = -\dot{\Phi} = \begin{cases} -1\,\mathrm{nV} & \text{for } \quad 0\,\mathrm{ms} < t < 1\,\mathrm{ms} \\ 1\,\mathrm{nV} & \text{for } \quad 20\,\mathrm{ms} < t < 21\,\mathrm{ms} \\ 0\,\mathrm{nV} & \text{else} \end{cases}$$

the maximal voltage is $\hat{U}_{\text{ind}} = 1\,\mathrm{nV}$.

(v) It can be measured for from 0 ms to 1 ms and from 20 ms to 21 ms .

(vi) While in the first case the current is flowing from the left to the right side (when looking in the direction of the magnetic field of the coil and the frame is entering the coil from above), in the second period it is in the opposite direction . (Lorentz force / Lenz's law)

2. Exercise: A train ride

Background: When trains drive along their railways their axles act as conductors which are moving through the magnetic field of the earth. Through induction a current will flow in the rails and the axles which will slow the train.

A metal crossbar is pulled with a constant velocity $v = 30\,\frac{\mathrm{m}}{\mathrm{s}}$ along two conducting rails placed at a distance $\ell = 1\,\mathrm{m}$, which are connected by a resistor $R = 50\,\mathrm{m}\Omega$ as shown on the next page. The crossbar and the rails have negligible resistance. A magnetic field $B = 50\,\mu\mathrm{T}$ is present at an angle of $\alpha = 60°$ to the rails.

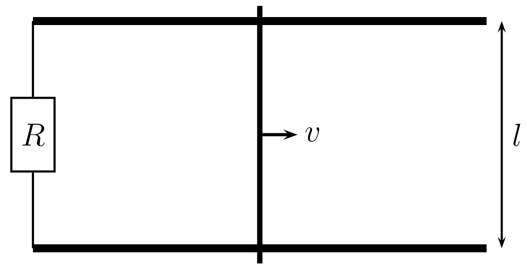
(i) Calculate the magnetic field perpendicular to the plane of the rails.

(ii) Determine the induced voltage U_{ind}.

(iii) Find the induction current I from Ohm's law.

(iv) What force F is needed to keep the metal crossbar at constant velocity v?

(v) Compare the power input by the force to the energy dissipated in the resistor.



Solution:

(i) The magnetic field responsible for induction is

$$\underline{\underline{B_\perp}} = B \sin\alpha = \underline{\underline{43\,\mu\text{T}}}$$

(ii) The induced voltage is given by

$$\underline{\underline{U_\text{ind}}} = B_\perp \ell v = \underline{\underline{1.3\,\text{mV}}}$$

(iii) From Ohm's law the induction current can be deduced to

$$\underline{\underline{I}} = \frac{U_\text{ind}}{R} = \underline{\underline{26\,\text{mA}}}$$

(iv) The force necessary to compensate induction can therefore be calculated to

$$\underline{\underline{F}} = I\ell B_\perp = \underline{\underline{1.1\,\mu\text{N}}}$$

(v) The power input by the force is given by the derivation of the gained energy when the crossbar is moving from x_1 at t_1 to a position x at t at the velocity v

$$\underline{\underline{P_F}} = \frac{\mathrm{d}W_F}{\mathrm{d}t} = \frac{\mathrm{d}\left(\int_{x_1}^{x} F\mathrm{d}x\right)}{\mathrm{d}t} = \frac{\mathrm{d}\left(\int_{t_1}^{t} Fv\mathrm{d}t\right)}{\mathrm{d}t} = Fv = \underline{\underline{34\,\mu\text{W}}}$$

This is equivalent to the power dissipated in the resistor:

$$\underline{\underline{P_R}} = U_\text{ind} I = \underline{\underline{34\,\mu\text{W}}}$$

3. Exercise: Magnetohydrodynamic generator

Background: Magnetohydrodynamic (MHD) generators are used in some power plants to increase efficiency, sometimes by 25%. Especially in old fossil-fuelled power plants the hot damps are first passed through a MHD generator and later to the usual generators.

A neutral stream, consisting of electrons and positively charged ions, the latter with mass $m_C = 2 \cdot 10^{-26}$ kg and charge $q_C = 12e$, is entering a parallel plane capacitor parallel to its plates with velocity $v = 9 \cdot 10^5 \, \frac{\mathrm{m}}{\mathrm{s}}$ on its central axis. Initially the capacitor with area $A = \ell^2 = 100 \, \mathrm{m}^2$ and distance $d = 1 \, \mathrm{cm}$ is not charged, but inside a homogeneous magnetic field of $B = 1 \, \mathrm{mT}$ perpendicular to the moving direction of the stream and parallel to the plates is applied. Neglect scattering within the stream.

(i) What kind of movement are the particles performing inside the capacitor?

(ii) At which two positions x_e, x_C will they hit the plates?

(iii) Which plate becomes negatively charged?

(iv) What voltage U_e should be applied so that all particles would pass the capacitor without been deflected.

Solution:

(i) Charged particle perform circular motion with radius

$$F_C = F_B \qquad \Rightarrow \qquad \frac{mv^2}{r} = qvB \qquad \Rightarrow \qquad r = \frac{mv}{qB}$$

when entering a magnetic field. Since this radius is $\underline{\underline{r_e = 5.119 \, \mathrm{mm}}}$ for electrons and $\underline{\underline{r_C = 9.4 \, \mathrm{m}}}$ for the ions they are captured by the capacitor. Thus they are moving in arcs towards the planes. After their collision they are neutral and will move linearly toward the end of the capacitor (the electron will be deflected several times).

(ii) From the Pythagorean theorem a formula for the position can be derivated to

$$r^2 = \left(r - \frac{d}{2}\right)^2 + x^2 \qquad \Rightarrow \qquad x = \sqrt{rd - \frac{d^2}{4}} = \frac{d}{2}\sqrt{4\frac{r}{d} - 1}$$

which states for electrons $\underline{\underline{x_e = 5.117 \, \mathrm{mm}}}$ and for the ions $\underline{\underline{x_C = 0.31 \, \mathrm{m}}}$.

(iii) When looking in the direction of the magnetic field and the stream is entering the capacitor from the left the <u>lower plate will be negatively</u> charged (Lorentz force).

(iv) All particles would pass the capacitor without deflection when the electrostatic force compensate the magnetostatic force:

$$F_E = F_B \Rightarrow q\frac{U}{d} = qvB \Rightarrow \underline{\underline{U}} = vBd = \underline{\underline{9 \, \mathrm{V}}}$$

Note:

- Mass of an electron: $m_e = 9.1 \cdot 10^{-31}$ kg
- Elementary charge: $e = 1.6 \cdot 10^{-19}$ C
- Permeability of vacuum: $\mu_0 = 1.26 \cdot 10^{-6} \, \frac{\mathrm{Vs}}{\mathrm{Am}}$