University Karlsruhe - International Department

Physics Course - Solutions Summer 2009

Stefan Kremer (kremer@tkm.uni-karlsruhe.de)
Holger Schmidt (hschmidt@tkm.uni-karlsruhe.de)

Revision sheet
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1. Exercise: Kinematics and Kinetics

(i) Give the $x(t)$ dependence for a uniform motion.

(ii) What is the $x(t)$ dependence for a motion with constant acceleration?

(iii) State and explain Newton's axioms.

(iv) Consider a mass point with mass m on an inclined plane (with angle α with respect to the horizontal). Give the force (due to gravity) acting on the body! What is its acceleration?

(v) Reconsider Atwood's machine with two masses m_1 and m_2. Give the acceleration of the system and the tension force!

Solution:

(i) $x(t) = vt + x_0$

(ii) $x(t) = \frac{1}{2}at^2 + v_0 t + x_0$

(iii) Newton's laws:

(a) Principle of inertia: $\sum_i \vec{F}_i = 0 \;\Leftrightarrow\; \vec{v} = const.$
A body on which no forces are acting will remain its state of movement.

(b) Principle of action: $\vec{F} = m\ddot{\vec{x}}$
The acceleration of a body is proportional to the force acting on it. The constant of the proportionality defines the mass.

(c) Actio is equal to reactio: $\vec{F}_{12} = -\vec{F}_{21}$
The force caused by one body on another one will have a counterpart on the first one caused by the second one.

(iv) Since the normal force $F_N = mg\cos\alpha$ will be counteracted by the plane only the downhill-slope force $F_D = mg\sin\alpha$ will causing a motion with constant acceleration $a = g\sin\alpha$.

(v) In Atwood's machine two masses m_1 and m_2 are connected by a rope. Since on the first mass will act the gravitational and tension force the ladder one will be $F_T = m_1(\ddot{y}_1 + g)$. In considering this force in Newtons second axiom of the second body the acceleration with which both masses are moving can be derived:

$$m_2\ddot{y}_1 = m_2 g - F_T = (m_2 - m_1)g - m_1\ddot{y}_1$$

$$\Rightarrow \qquad \ddot{y}_1 = \frac{m_2 - m_1}{m_1 + m_2}g, \qquad F_T = 2g\frac{m_1 m_2}{m_1 + m_2}$$

2. Exercise: Energy and Collisions

(i) What is energy conservation?

(ii) How can the velocity from the kinetic energy be obtained?

(iii) At what height h will a mass m stop on an inclined plane when it is moving towards it with velocity v?

(iv) State the energy of a compressed spring.

(v) How is in general a total elastic collision defined? Name and write down the two corresponding conservation laws.

(vi) Give in general the conditions for a total inelastic collision.

(vii) Simplify these for the case that one mass is at rest before the collision and the masses stick together afterwards.

Solution:

(i) That the total energy, the sum of kinetic and potential energy, of the whole system will be the same at any time.

(ii) $E_{\text{kin}} = \frac{1}{2}mv^2 \qquad \Rightarrow \qquad v = \sqrt{\frac{2E_{\text{kin}}}{m}}$

(iii) E.C.: $\frac{1}{2}mv^2 = mgh \qquad \Rightarrow \qquad h = \frac{v^2}{2g}$

(iv) $E_{\text{spring}} = \frac{1}{2}Dx^2$ (x is the distance of the compression)

(v) In a total elastic collision both momentum and energy are conserved, e.g. for two particles:

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1u_1 + m_2u_2 \\ \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 &= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \end{aligned}$$

where v_k is the velocity of the k particle with mass m_k before and u_k the velocity after the collision.

(vi) In a total inelastic collision momentum is still conserved, but some energy is converted into heat Q:

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1u_1 + m_2u_2 \\ \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 &= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 + Q \end{aligned}$$

(vii) For the inelastic collision where the first mass is at rest before the collision they read

$$\begin{aligned} m_2v_2 &= (m_1 + m_2)u \\ \frac{1}{2}m_2v_2^2 &= \frac{1}{2}(m_1 + m_2)u^2 + Q \end{aligned}$$

and therefore

$$u = \frac{m_2}{m_1 + m_2}v_2, \qquad Q = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}v_2^2$$

3. Exercise: Rigid body and Oscillations

(i) How is the moment of inertia of a rigid body defined?

(ii) Derive the moment of inertia of a uniform rod of length ℓ and mass m with respect to an axis of rotation through the centre.

(iii) State the energy of a rotating rigid body.

(iv) Give the parallel axis theorem.

(v) Write down the equation of motion for an harmonic oscillator and give the general solution.

(vi) State the equations of motion for a simple pendulum and a physical pendulum.

(vii) Specify both equations for small angles ϕ and give the solutions.

(viii) What are the periods of the oscillations?

Solution:

(i) $\Theta = \rho \int_V \vec{r}^{\,2} \mathrm{d}V$

(ii) $\Theta_{\text{rod}} = \frac{m}{\ell} \int_{-\ell/2}^{\ell/2} \vec{x}^{\,2} \mathrm{d}x = \frac{m}{\ell} \frac{1}{3} \left(\left(\frac{\ell}{2}\right)^3 - \left(-\frac{\ell}{2}\right)^3 \right) = \frac{1}{12} m\ell^2$

(iii) $E_{\text{rot}} = \frac{1}{2} \Theta \omega^2$

(iv) $\Theta = \Theta_{\text{cm}} + ma^2$ where the new moment of inertia Θ is through an axis parallel to the one of the centre of mass (cm) at an distance a.

(v) $\ddot{x} + \omega^2 x = 0 \qquad \Rightarrow \qquad x(t) = A \sin(\omega t + \phi)$

(vi) For simple pendulum it reads $\ell \mathrm{d}\alpha + g \sin\alpha = 0$ and for the physical pendulum it is $\Theta \ddot{\alpha} + mg\ell\alpha = 0$.

(vii) For small angles they read $\ddot{\alpha} + \frac{g}{\ell}\alpha = 0$ and $\ddot{\alpha} + \frac{mg\ell}{\Theta}\alpha = 0$

(viii) Therefore the periods are $T_{\text{math}} = 2\pi\sqrt{\frac{\ell}{g}}$ and $T_{\text{phys}} = 2\pi\sqrt{\frac{\Theta}{gm\ell}}$

4. Exercise: Electrostatics

(i) What is the electric field of point charge Q and a capacitor with parallel plates of area ℓ^2, charged by a voltage U?

(ii) State the electric force $\vec{F}$ on a charge q in an electric field $\vec{E}$.

(iii) How is the electric field E related to the voltage U and the distance d of the plates of a parallel plate capacitor?

(iv) What kind of motion does a charge q if it enters a parallel plate capacitor perpendicular to the electric field? Give explicit the $\vec{r}(t)$ and $\vec{v}(t)$ dependence and the geometrical path $y(x)$ for that kind of motion!

(v) How can the angle be obtained with which such a charge will leave the capacitor?

(vi) State the definition of the capacitance C.

(vii) What is the capacitance of parallel plate capacitor?

(viii) What energy does a charge q gain in such a parallel plate capacitor (i.e. if the capacitor is used as an electron source)?

Solution:

(i) Point charge: $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
Parallel plate capacitor: $E = \frac{Q}{\epsilon_0 \ell^2}$

(ii) $\vec{F}_e = q\vec{E}$

(iii) $E = \frac{U}{d}$

(iv) It performs a parabola:

$$\vec{v}(t) = \begin{pmatrix} v_0 \\ \frac{F_e}{m}t \\ 0 \end{pmatrix}$$

$$\vec{r}(t) = \begin{pmatrix} v_0 t \\ \frac{1}{2}\frac{F_e}{m}t^2 \\ 0 \end{pmatrix}$$

$$y(x) = \frac{F_e}{2mv_0^2}x^2$$

(v) $\tan\alpha = \frac{v_y}{v_x} \quad \Rightarrow \quad \alpha = \arctan\frac{F_e \ell}{m v_0^2}$

(vi) $C = \frac{Q}{U}$

(vii) $C = \epsilon_0 \epsilon_r \frac{\ell^2}{d}$

(viii) $W = qU$

5. **Exercise: Electromagnetism**

(i) State and explain the content of Maxwell's equations.

(ii) What is the magnetic field of a coil and a wire?

(iii) How is the magnetic flux Φ defined?

(iv) What is the Lorentz-Force, i.e. the force of the magnetic field $\vec{B}$ on a charge q, that moves with velocity $\vec{v}$?

(v) How can one calculate the force of a magnetic field B on a wire of length ℓ carrying a current I?

(vi) What kind of path describes a charged particle in homogeneous magnetic field $\vec{B}$?

(vii) Explain the function of a velocity selector.

Solution:

(i) Maxwell's equations:

(a) Gauss' law: $\oint_S \vec{E}\mathrm{d}\vec{A} = \frac{Q_{\text{inside}}}{\epsilon_0}$

If the summation of the electric field around a closed surface is not vanishing, there have to be charges inside it generating the electric field.

(b) Gauss' law for magnetism: $\oint_S \vec{B}\mathrm{d}\vec{A} = 0$

The magnetic field is not generated by some kind of (resting) charges. (The magnetic fieldlines are closed.)

(c) Faraday's law of induction: $\oint_C \vec{E}\mathrm{d}\vec{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_S \vec{B}\mathrm{d}\vec{A}$

If the magnetic field in an area (or more precise the magnetic flux) is changing with time, it will cause an voltage around the edge of such a surface.

(d) Ampere's law: $\oint_C \vec{B}\mathrm{d}\vec{\ell} = \mu_0 \int_S \vec{j}\mathrm{d}\vec{A} + \frac{\mathrm{d}}{\mathrm{d}t}\int_S \vec{E}\mathrm{d}\vec{A}$

A magnetic field is generated around a current. As seen in a circuit with a capacitor, where the initial current is replaced by a changing electric field, also such a change can result in a magnetic field.

(ii) Wire: $B = \frac{\mu_0 I}{2\pi r}$

Coil: $B = \mu_0 \frac{N}{\ell} I$

(iii) $\Phi = \int \vec{B}\mathrm{d}\vec{A}$

(iv) $\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B})$

(v) $F = I\ell B_\perp$

(vi) It would describe a circular motion.

(vii) When putting an electrostatic force opposite to a magnetic force only one velocity $v = \frac{E}{B}$ would not be deflected.

6. Exercise: Circuits

(i) What is the content of the Kirchhoff rules?

(ii) State Ohm's law.

(iii) What is the power dissipated in a resistor if a voltage U is applied?

(iv) How can the equivalent resistance of two parallel resistors be calculated?

(v) What is the equivalent resistance of two serial connected resistors?

(vi) How can a battery with voltage U_0 and internal resistance r be expressed in a equivalent circuit?

(vii) What is period of an RCL-circuit?

(viii) Sketch the $I(t)$ dependence of the opening and closing of a RL circuit.

(ix) How is the charge depending on time if an RC circuit is opened or closed?

Solution:

(i) The directed sum of the current in each point and the directed sum of the voltage in each circle vanishes $\sum_k I_k = 0,\ \sum_k U_k = 0$.

(ii) $R = \frac{U}{I}$

(iii) $P = UI = \frac{U^2}{R}$

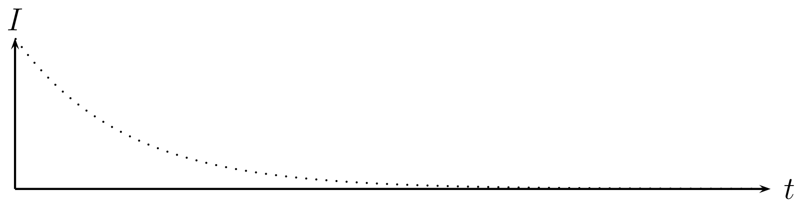
(iv) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

(v) $R_{eq} = R_1 + R_2$

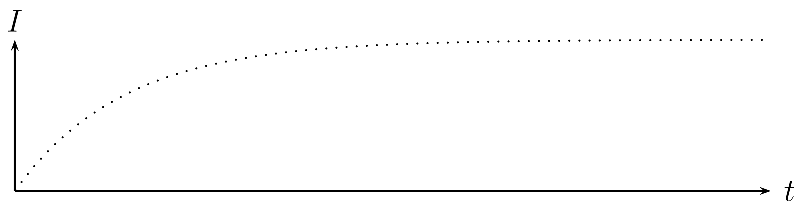
(vi) As voltage source with U_0 and the inner resistance r in series connection.

(vii) $T = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$

(viii) Opening: Exponential decay of the current:



Closing: Negative exponential decay from zero to the maximal current $\frac{U_0}{R}$.



(ix) Opening: $Q(t) = Q_0 \mathrm{e}^{-t/\tau}$ where the time constant is $\tau = RC$

Closing: $Q(t) = CU_0 \left(1 - \mathrm{e}^{-t/\tau}\right)$