UNIVERSITY KARLSRUHE - INTERNATIONAL DEPARTMENT

Physics Course - Solutions   Summer 2009

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Extra Tutorial
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1. Exercise

(i) Give two different possible SI-units of the electric field.

(ii) What is the electric field of a point charge q?

(iii) What is the electric force $\vec{F}$ on a charge q if the electric field $\vec{E}$ is given?

(iv) What is the relation between the electric field E, the voltage U and the distance d of the plates of a parallel plate capacitor?

(v) What kind of motion does a charge q if it enters a parallel plate capacitor parallel to the electric field?

(vi) What kind of motion does a charge q if it enters a parallel plate capacitor perpendicular to the electric field?

(vii) How is the capacitance C defined?

(viii) What is the capacitance of parallel plate capacitor.
Distance between the plates: d. Area of the plates: A.

(ix) What energy does a charge q gain in a parallel plate capacitor with an applied voltage of U (i.e. if the capacitor is used as an electron source)?

(x) What is the physical effect of a dielectric? How does it affect the capacitance?

(xi) What is the content of the Kirchhoff rules?

(xii) What states Ohm's law? What is the (microscopic) starting point to derive this law?

(xiii) A metallic wire ($\rho_s = 10^{-5}\,\Omega\,\mathrm{m}$) has cross section $A = 0.5\,\mathrm{mm}$ and length $\ell = 2\,\mathrm{m}$. How are the current I and voltage U related in this wire?

(xiv) What is the power dissipated in a resistor if a voltage U is applied?

Solution:

(i) $[\vec{E}] = \frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{N}}{\mathrm{C}} = \frac{\mathrm{kg\,m}}{\mathrm{C\,s^2}} = \frac{\mathrm{kg\,m}}{\mathrm{A\,s^3}} = \frac{\mathrm{C}}{\mathrm{F\,m}}$

(ii) $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\vec{e}_r$

(iii) $\vec{F} = q\vec{E}$

(iv) $E = \frac{U}{d}$

(v) Linear motion with constant acceleration

(vi) Deviation with constant acceleration

(vii) $C = \frac{Q}{U}$

(viii) $C = \epsilon_0 \frac{A}{d}$

(ix) $W = qU$

(x) Dipoles within the dielectric are orientated to weaken the applied electric field. This leads to an increase of the capacitance.

(xi) The directed sum of all currents vanishes in each point (charge/current conservation). The directed sum of all voltages vanishes in each circle (energy conservation).

(xii) Ohm's law states that the voltage drop at a resistor is proportional to the current: $R = \frac{U}{I}$. It was derived by replacing the scattered motion of a charge in a wire at an applied voltage U by a Stokes friction term in the classical equation of motion. The static solution gave microscopic version of Ohm's law.

(xiii) They are linear dependend: $U = RI = \rho_s \frac{\ell}{A} I = 80\,\frac{\mathrm{V}}{\mathrm{A}}\,I$

(xiv) $P = UI = \frac{1}{R} U^2$

2. Exercise

Consider a simplified version of Millikan's oildrop experiment. (The experiment with which the electric charge was measured for the first time.) A uniform electric field is provided by a pair of horizontal parallel plates of area A with distance d and a high potential difference U between them. A charged drop of oil with mass m is allowed to drift in between them (i.e. it is exposed to the gravitational force). By varying the potential, the drop can be made to stay steady. Determine the charge Q of the oil drop.

Solution:
In the steady state the gravitational force has to be compensated by the electrostatic force:

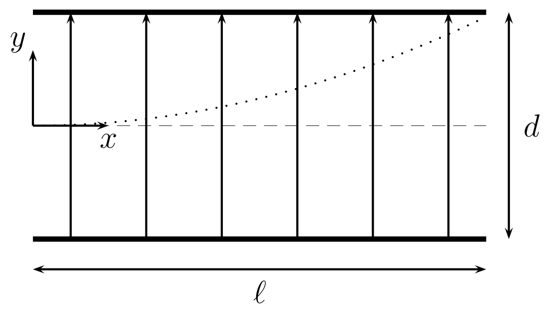
$$F_g = F_{el} \qquad \Rightarrow \qquad mg = QE = Q\frac{U}{d} \qquad \Rightarrow \qquad \underline{\underline{Q = \frac{mgd}{U}}}$$

3. Exercise

Consider a point charge $q = e$ with mass $m = 10^{-16}\,\mathrm{kg}$ which is accelerated by crossing a potential difference $\Delta U = 1250\,\mathrm{V}$. Then it moves to the right along the axis of the square parallel plate capacitor as shown. The plates of the parallel plate capacitor are separated by a distance $d = 1\,\mathrm{cm}$, have length $\ell = 3\,\mathrm{cm}$. There is an electric field $E = 2 \cdot 10^4\,\frac{\mathrm{V}}{\mathrm{m}}$ in the region between the plates.

(i) What is the velocity v_0 of the charge before it reaches the deflection plates?

(ii) Write down the position of the particle in the parallel plate capacitor either as vector $\vec{r}(t)$ or in component form (i.e. $x(t)$ and $y(t)$).



(iii) How long does the charge need to pass the parallel plate capacitor?

(iv) Find the angle with respect to the horizontal when the point charge leaves the parallel plate capacitor.

(v) What potential difference U have to be applied when the point charge should leave the capacitor at the upper right corner.

There is a screen separated at a distance $s = 12\,\mathrm{cm}$ from the end of the plates.

(vi) At what distance Δy from the axis will the charge strike the screen?

Solution:

(i) From energy conservation follows

$$W_{kin} = E_e \qquad \Rightarrow \qquad \frac{1}{2}mv_0^2 = e\Delta U \qquad \Rightarrow \qquad \underline{\underline{v_0}} = \sqrt{\frac{2e\Delta U}{m}} = \underline{\underline{2\,\frac{\mathrm{m}}{\mathrm{s}}}}\,.$$

(ii) For a deviation with constant acceleration the position can be calculated to

$$\underline{\underline{\vec{r}(t)}} = \begin{pmatrix} v_0 t \\ \frac{1}{2}\frac{F_e}{m}t^2 \end{pmatrix} = \begin{pmatrix} v_0 t \\ \frac{1}{2}\frac{eE}{m}t^2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2\,\frac{\mathrm{m}}{\mathrm{s}}\,t \\ 16\,\frac{\mathrm{m}}{\mathrm{s}^2}t^2 \end{pmatrix}}}\,.$$

(iii) The transition time is given by

$$\underline{\underline{t_0}} = \frac{\ell}{v_0} = \underline{\underline{15\,\mathrm{ms}}}\,.$$

(iv) Since $\vec{v}(t) = \dot{\vec{r}}(t) = \begin{pmatrix} v_0 \\ \frac{eE}{m}t_0 \end{pmatrix}$ the angle can be determined by

$$\tan\alpha = \frac{v_y}{v_x} = \frac{eEt_0}{mv_0} = 0.24 \qquad \Rightarrow \qquad \underline{\underline{\alpha = 13.5^\circ = 0.236}}\,.$$

(v) Due to the position have to be $\frac{d}{2}$ after the transition of the capacitor, the voltage follows to

$$\frac{d}{2} = \Delta y_{cap} = r_y(t_0) = \frac{1}{2}\frac{eE}{m}t_0^2 = \frac{1}{2}\frac{eU}{md}\frac{\ell}{v_0^2} \qquad \Rightarrow \qquad \underline{\underline{U}} = \frac{md^2v_0^2}{e\ell^2} = \underline{\underline{278\,\mathrm{V}}}\,.$$

(vi) The distance Δy is the sum of the deviation in the capacitor and y-motion Δy_s outside it. The latter can be determined by the triangle with anlge α and the sides s and Δy_s at the right angle:

$$\Delta y = \Delta y_{cap} + \Delta y_s = \frac{1}{2}\frac{eE}{m}t_0^2 + s\tan\alpha = 32.4\,\mathrm{mm}$$

4. Exercise

The plates of a parallel-plate capacitor are $d = 3.3\,\mathrm{mm}$ apart, each has an area of $A = 12\,\mathrm{cm}^2$. Each plate carries a charge of magnitude $Q = 4.4 \cdot 10^{-8}\,\mathrm{C}$. The plates are in vacuum.

(i) What is the capacitance C?

(ii) What is the potential difference U between the plates?

The capacitor is not connected to any battery. A dielectric with a dielectric constant of $\epsilon_r = 2$ is inserted into the capacitor, filling the total gap.

(iii) How does this influence the charge on the plates?

(iv) How does this influence the potential difference between the plates?

The capacitor is connected to a battery. Again a dielectric with a dielectric constant of $\epsilon_r = 2$ is inserted into the capacitor, filling the total gap.

(v) How does this influence the charge on the plates?

(vi) How does this influence the potential difference between the plates?

Solution:

(i) The capacitance is given by

$$\underline{\underline{C}} = \epsilon_0 \frac{A}{d} = \underline{\underline{3.2\,\mathrm{pF}}}$$

(ii) From the definition of the capacitance the voltage follows to

$$C = \frac{Q}{U} \qquad \Rightarrow \qquad \underline{\underline{U}} = \frac{Q}{C} = \underline{\underline{14\,\mathrm{kV}}}\,.$$

(iii) Because it is not connected to a battery, charges are conserved. Therefore the charge does $\underline{\underline{\text{not change}}}$.

(iv) Since only the voltage can change it will be reduced to

$$C_r = \frac{Q}{U_r} \qquad \Rightarrow \qquad \underline{\underline{U_r}} = \frac{Q}{C_r} = \frac{1}{\epsilon_r}\frac{Q}{C} = \frac{1}{\epsilon_r}U = \underline{\underline{6.8\,\mathrm{kV}}}\,.$$

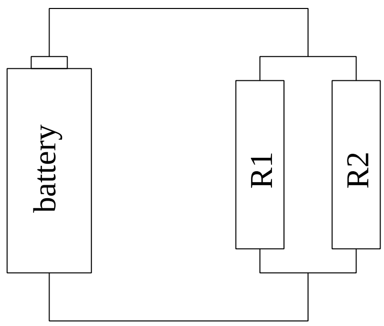
(v) When the voltage is kept fixed by the battery only the charge can change to

$$C_r = \frac{Q}{U_r} \qquad \Rightarrow \qquad \underline{\underline{Q}} = UC_r = \epsilon_r UC = \epsilon_r Q = \underline{\underline{8.8 \cdot 10^{-8}\,\mathrm{C}}} \,.$$

(vi) Due to the capacitor lies at the battery it lies still at the $\underline{\underline{\text{same voltage}}}$.

5. Exercise

Consider a battery with a voltage $U = 5\,\mathrm{V}$ and an inner resistance of $R_b = 1\,\Omega$. It is connected to two parallel connected resistors with resistances $R_1 = 40\,\Omega$ and $R_2 = 60\,\Omega$.



(i) Calculate the equivalent resistance R_R of the two resistors.

(ii) Calculate the resistance R_{eq} of the whole circuit.

(iii) Calculate the current I in the circuit and the power P provided by the battery.

(iv) Calculate the voltage U_R that falls at the two resistors and the voltage U_b that falls at the battery.

(v) Calculate the currents I_1 and I_2 through the resistors R_1 and R_2.

(vi) Which resistor gets hotter and why?

Solution:

(i) The equivalent resistance of two parallel resistors is given by

$$\underline{\underline{R_R}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \underline{\underline{24\,\Omega}} \,.$$

(ii) The total resistance of the whole circuit is a series connection between this resistance and the inner resistance of the battery:

$$\underline{\underline{R_{eq}}} = R_R + R_b = \underline{\underline{25\,\Omega}}$$

(iii) The current can be derived from Ohm's law:

$$R_{eq} = \frac{U}{I} \qquad \Rightarrow \qquad \underline{\underline{I}} = \frac{U}{R_{eq}} = \underline{\underline{200\,\mathrm{mA}}}$$

From it the power provided by the battery can be determined to

$$\underline{\underline{P}} = (U - U_b)I = UI - R_b I^2 = \underline{\underline{960\,\mathrm{mW}}}\,.$$

(iv) At the inner resistance falls a voltage of

$$\underline{\underline{U_b}} = R_b I = \underline{\underline{200\,\mathrm{mV}}}\,,$$

while at both resistors the potential difference amounts to

$$\underline{\underline{U_R}} = R_R I = \underline{\underline{4.8\,\mathrm{V}}}\,.$$

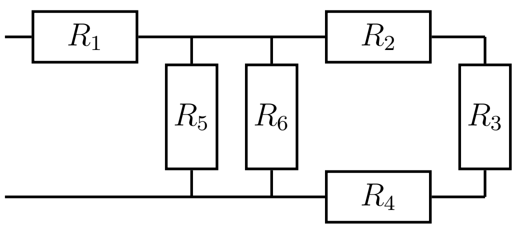
(v) The currents can be evaluated by Ohm's law to

$$\underline{\underline{I_1}} = \frac{U_R}{R_1} = \underline{\underline{120\,\mathrm{mA}}} \qquad \underline{\underline{I_2}} = \frac{U_R}{R_2} = \underline{\underline{80\,\mathrm{mA}}}\,.$$

(vi) Due to $P_1 = U_R I_1 = 576\,\mathrm{mW} \;>\; 384\,\mathrm{mW} = U_R I_2 = P_2$ the $\underline{\underline{\text{left resistor } (R_1)}}$ gets hotter (more energy is dissipated per second).

6. Exercise

Calculate the equivalent resistance of the following resistor network with $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\,\Omega$, $R_4 = 5\,\Omega$, $R_5 = 4\,\Omega$, $R_6 = 6\,\Omega$:



Solution: The three resistors on the right sides form a series resistance of

$$R_\alpha = R_2 + R_3 + R_4 = 10\,\Omega$$

which is parallel connected to R_5 and R_6 forming a equivalent resistance of

$$R_\beta = \frac{1}{\frac{1}{R_5} + \frac{1}{R_6} + \frac{1}{R_\alpha}} = \frac{R_5 R_6 R_\alpha}{R_5 R_6 + R_5 R_\alpha + R_6 R_\alpha} = \frac{60}{31}\,\Omega \approx 1.9\,\Omega$$

This resistance is then serial connected to R_1 which gives a total resistance of

$$\underline{\underline{R_{eq}}} = R_1 + R_\beta = R_1 + \frac{(R_2 + R_3 + R_4)R_5 R_6}{R_5 R_6 + (R_5 + R_6)(R_2 + R_3 + R_4)} = \frac{91}{31}\,\Omega \approx \underline{\underline{2.94\,\Omega}}$$

7. Exercise*

Calculate the equivalent resistance of a cube consisting of resistors of $R = 1\Omega$ at the edges which is contacted at two opposite corners.

Solution: In connecting points on the same potential (same voltage drop after going one resistor in each direction) an equivalent circuit can be obtained containing a series connection of three parallel, six parallel and again three parallel resistors. The equivalent resistance is therefore given by

$$\underline{\underline{R_{eq}}} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6}\Omega = \frac{5}{6}\,\Omega \approx \underline{\underline{0.83\,\Omega}}$$

8. Exercise*

A metal cube with dimension $\ell \times \ell \times \Delta$ is fully inserted at position x into a capacitor with plates of size $A = \ell^2$ at a distance d which is charged by a voltage U. What is the equivalent capacitance and how much energy is needed for in this process?

Solution: The equivalent circuit is a parallel connection of capacitors with reduced distances x and $d - \Delta - x$ leading to a total capacitance of

$$\underline{\underline{C}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \epsilon_0 A \frac{1}{d - \Delta - x + x} = \underline{\underline{\epsilon_0 \frac{A}{d - \Delta}}} \, .$$

The energy can therefore determined by

$$\begin{aligned} \underline{\underline{\Delta W}} &= \frac{1}{2}CU^2 - \frac{1}{2}C_{empty}U^2 = \frac{1}{2}U^2(C - C_{empty}) = \frac{1}{2}U^2\epsilon_0 A\left(\frac{1}{d - \Delta} - \frac{1}{d}\right) \\ &= \frac{1}{2}U^2\epsilon_0 A\frac{d - d + \Delta}{d(d - \Delta)} = \underline{\underline{\frac{1}{2}U^2\epsilon_0 A\frac{\Delta}{d(d - \Delta)}}} \end{aligned}$$

Note:

- Permittivity of free space: $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\mathrm{C}}{\mathrm{Vm}}$
- Elementary charge: $e = 1.6 \cdot 10^{-19}\,\mathrm{C}$
- General definition of voltage

$$U = \int_{x_1}^{x_2} E \,\mathrm{d}x$$

- Ohm's law

$$j = \sigma E$$

- Current and voltage in a wire of length ℓ and cross section A

$$I = jA, \quad U = E\ell$$

- Resistance

$$R = \rho_s \frac{\ell}{A}$$

- Electric power

$$P = UI$$