University Karlsruhe - International Department

Physics Course - Solutions   Summer 2009

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Extra Tutorial
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1. Exercise

(i) State Ohm's law.

(ii) What is the content of Kirchhoff's rules?

(iii) What is the equivalent resistance of two parallel resistors with resistance R_1 and R_2?

(iv) What is the equivalent resistance of two serial connected resistors with resistance R_1 and R_2?

(v) How can a battery with voltage U_0 and internal resistance r be expressed in a equivalent circuit?

(vi) How is the magnetic field of a wire connected to the current flowing inside it?

(vii) State the magnetic field of a coil, with length ℓ and turns N surrounding a volume $V = \ell \cdot A$, through which flows a current I.

(viii) What is the Lorentz-Force of a magnetic field $\vec{B}$ on a charge q, that moves with velocity $\vec{v}$? Give the vectorial and the scalar expression!

(ix) Find the force $\vec{F}$ on a charge q that moves with a velocity $\vec{v}$ in a capacity, charged by a voltage U, and where a magnetic field $\vec{B}$ is present.

(x) How can one calculate the force of a magnetic field $\vec{B}$ on a wire of length ℓ which carries a current I.

(xi) What kind of path describes a charged particle in homogeneous magnetic field $\vec{B}$.

(xii) What is a velocity selector and how does it work?

(xiii) How is the magnetic flux Φ defined?

(xiv) What is the content of Faraday's induction law, i.e. how is the magnetic flux Φ related to the induced voltage U_{ind}?

(xv) State and explain Lenz's law as equation.

(xvi) How is the self-inductance of a coil related to its geometric quantities?

(xvii) What kind of voltage is provided by an electric generator in contrast to a battery?

Solution:

(i) $R = \frac{U}{I}$

(ii) Conservation of voltage in each closed cycle $\sum_k U_k = 0\,\mathrm{V}$ and conservation of current in each point $\sum_k I_k = 0\,\mathrm{A}$ is stated by Kirchhoff's rules.

(iii) $R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

(iv) $R_{eq} = R_1 + R_2$

(v) By a series connection of a voltage source and an inner resistance the battery can be modeled.

(vi) $B = \frac{\mu_0}{2\pi r} I$

(vii) $B = \mu_0 \frac{N}{\ell} I$

(viii) $\vec{F_L} = q\vec{v} \times \vec{B} \qquad \Rightarrow \qquad F_L = qvB$

(ix) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q(\frac{U}{d}\vec{e}_d + \vec{v} \times \vec{B})$

(x) $F = \ell IB$

(xi) The charged particle in a homogeneous field describes a circle.

(xii) The velocity selector let only passes particles with a certain velocity undeflected. It uses the balance of the electrostatic and magnetic force in order to fix the velocity that passes undeflected to the ratio of both fields.

(xiii) $\Phi = \int \vec{B} \mathrm{d}\vec{A}$

(xiv) Faraday's induction law states that a changing magnetic flux would produce an electric field:

$$U_{ind} = -\dot{\Phi}$$

(xv) Lenz's law is the minus sign in the above formula (or in the formula $U_{ind} = -L\dot{I}$). It states that the action which leads to induction will be (partly) counteracted by the induction.

(xvi) $L = \mu_0 \frac{N^2}{\ell}$

(xvii) Electric generators provides Alternating Current (A.C.) while at batteries Direct Current (D.C.) is usually measured.

2. Exercise

A metallic rod with resistance $R_r = 1.5\,\Omega$ and mass $m = 150\,\mathrm{g}$ lies on two rails placed at a distance $\ell = 1\,\mathrm{m}$ where a perpendicular magnetic field $B = 200\,\mathrm{mT}$ into the ground is applied. The rails are connected at one end by a battery with voltage $U_0 = 3\,\mathrm{V}$ and inner resistance $r = 500\,\mathrm{m}\Omega$.

At first neglect self-induction effects:

(i) Draw the equivalent circuit.

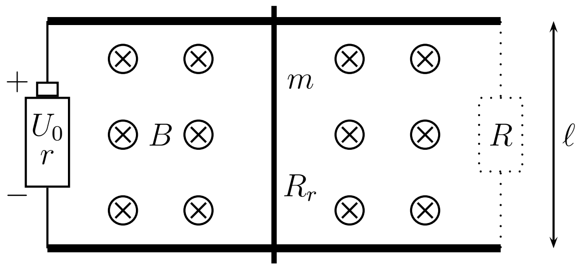
(ii) State the equivalent resistance R_{eq} of the circuit.

(iii) What is the current I through the rod?

(iv) Calculate the force F on the rod.

(v) In which direction does the rod moves?

(vi) State the position depending on time.



Let now be a resistance of $R = 3\,\Omega$ between the ends of the rails opposite to the battery.

(vii) Draw the equivalent circuit.

(viii) What is now the equivalent resistance R'_{eq} which lies at the battery?

(ix) What current I' flows now through the rod?

(x) What would be the velocity after $t = 3\,\mathrm{ms}$?

Now consider again the original situation with only the battery and the rod but look at the terminal situation with self-induction effects.

(xi) How large has to be the induced current I_{ind} to compensate the force F in (iv)?

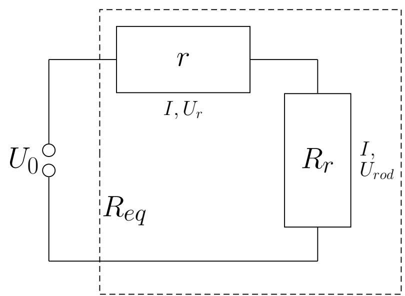
(xii) Compare this current with the one flowing in the circuit without self-induction effects, calculated in (iii). What would be the total current I_{tot}?

(xiii) How depend the induced voltage U_{ind} in the rod from its velocity v?

(xiv) Use Kirchhoff's voltage law (and Lenz's law) to determine the terminal velocity.

Solution:

(i)



(ii) Since the rod and the internal resistance are forming a series connection the equivalent resistance is given by

$$\underline{\underline{R_{eq}}} = r + R_r = \underline{\underline{2\,\Omega}}\,.$$

(iii) The current through the circuit is determined by Ohm's law:

$$R_{eq} = \frac{U_0}{I} \qquad \Rightarrow \qquad \underline{\underline{I}} = \frac{U_0}{R_{eq}} = \underline{\underline{1.5\,\mathrm{A}}}$$

(iv) When this current is flowing through the rod it causes a Lorenz force of

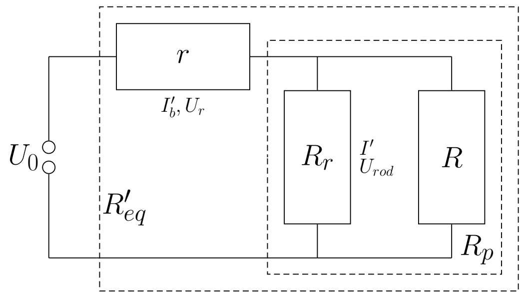
$$\underline{\underline{F}} = IB\ell = \underline{\underline{0.3\,\mathrm{N}}}\,.$$

(v) Since the current flows from the positive end of the battery to the negative one it flows downwards in the rod. From the three finger rule it can then be deduced that the rod will move <u>to the right</u>, away from the resistor.

(vi) Neglecting self-inductance the motion is one with constant acceleration:

$$\underline{\underline{x(t)}} = \frac{1}{2}\frac{F}{m}t^2 = \underline{\underline{1\,\frac{\mathrm{m}}{\mathrm{s}^2}\cdot t^2}}$$

(vii)



(viii) Now the rod and the new resistance are forming a parallel connection with resistance

$$R_p = \frac{1}{\frac{1}{R_r} + \frac{1}{R}} = \frac{R_r R}{R_r + R} = 1\,\Omega$$

which is then put in series with the internal resistance leading to a total resistance of

$$\underline{\underline{R_{eq}'}} = r + R_p = \underline{\underline{1.5\,\Omega}}$$

at the battery.

(ix) In order to calculate the current in the rod the one at the battery first has to be determined by Ohm's law to

$$I_b' = \frac{U_0}{R_{eq}'} = 2\,\mathrm{A}$$

from which the voltage at the rod can be found by Kirchhoff's voltage law/Ohm's law to be

$$U_{rod} = U_0 - U_r = U_0 - rI_b' = 2\,\mathrm{V}$$

which finally leads to the current in the rod by Ohm's law:

$$\underline{\underline{I'}} = \frac{U_{rod}}{R_r} = \underline{\underline{1.3\,\mathrm{A}}}$$

(x) Since it is again a linear motion with constant acceleration of

$$a = \frac{F}{m} = \frac{I'B\ell}{m} = 1.7\,\frac{\text{m}}{\text{s}^2}$$

the velocity after $t = 3\,\text{ms}$ will be

$$\underline{\underline{v(t = 3\,\text{ms})}} = at = \underline{\underline{5.2\,\frac{\text{mm}}{\text{s}}}}\,.$$

(xi) The force the induced current would produce is given by $F = I_{ind}B\ell$ and therefore the current has to be

$$\underline{\underline{I_{ind}}} = \frac{F}{B\ell} = \underline{\underline{1.5\,\text{A}}}\,.$$

(xii) Since this current is equal to the one given by the battery but in the opposite direction (Lenz's law) the total current would vanish

$$\underline{\underline{I_{tot} = 0\,\text{A}}}\,.$$

(If a total current would flow a total force would occur which would then not be the steady state.)

(xiii) The induced voltage can be derived by the flux which is given by

$$\Phi = \int \vec{B}\mathrm{d}\vec{A} = BA = B\ell x(t)$$

and therefore the induced voltage can be obtained to be

$$\underline{\underline{U_{ind}}} = \dot{\Phi} = B\ell\dot{x} = B\ell v = \underline{\underline{0.2\,\text{Tm}\cdot v}}\,.$$

(xiv) Since the voltage provided by the rod is opposite to the applied voltage, Kirchhoff's voltage law reads

$$U_0 = U_{ind} + U_r + U_{rod} = B\ell v + R_{eq}I_{tot}$$

and due to the fact that the terminal current vanishes the velocity can be calculated to

$$\underline{\underline{v}} = \frac{U_0}{B\ell} = \underline{\underline{15\,\frac{\text{m}}{\text{s}}}}\,.$$