University Karlsruhe - International Department

Physics Course - Solutions Summer 2009

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Extra Tutorial
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1. Exercise

Consider two masses $m_1 > m_2$ on opposite sites of a roof with angle α with respect to the ground. Both masses are at height h and are connected by a robe over the top of the roof. Both masses are exposed to gravitational force ($g = 10\,\frac{\text{m}}{\text{s}^2}$).

(i) Determine the acceleration of the system.

(ii) Find the time t_1 the mass m_1 hits the ground.

(iii) What will be the tension force in the rope?

(iv) Calculate these quantities for $\alpha = 30°$, $m_1 = 8\,\text{kg}$, $m_2 = 4\,\text{kg}$ and $h = 10\,\text{m}$.

Solution:

(i) $(m_1 + m_2)a = (m_1 - m_2)g\sin\alpha \qquad \Rightarrow \qquad \underline{\underline{a = \frac{m_1-m_2}{m_1+m_2} g\sin\alpha}}$

(ii) $0 = \frac{h}{\sin\alpha} - \frac{1}{2}at_g^2 \qquad \Rightarrow \qquad \underline{\underline{t_g}} = \sqrt{\frac{2h}{a\sin\alpha}} = \underline{\underline{\sqrt{\frac{2(m_1+m_2)h}{(m_1-m_2)g\sin^2\alpha}}}}$

(iii) $m_1 a = m_1 g\sin\alpha - F_T \qquad \Rightarrow \qquad \underline{\underline{F_T}} = m_1(g\sin\alpha - a) = \underline{\underline{\frac{2m_1m_2}{m_1+m_2} g\sin\alpha}}$

(iv) $\underline{\underline{a = 1.7\,\frac{\text{m}}{\text{s}^2}}} \qquad \underline{\underline{t_g = 4.9\,\text{s}}} \qquad \underline{\underline{F_T = 27\,\text{N}}}$

2. Exercise

A uniform rod of mass m and length L is hung at a distance $\frac{L}{4}$ of one of its ends and is initially pulled back at an angle $\phi = \phi_0$ to the vertical. The rod is exposed to gravity. Vertical below the attachment a spring with constant D is placed.

(i) What is the moment of inertia of the system?

(ii) Determine the angular velocity when the rod reaches the vertical position.

(iii) At this position it will hit the spring. What is the maximum compression s of the spring?

(iv) Now the spring is removed. State the equation of motion and give the general solution for small angles ϕ.

(v) How long does it take for the rod to reach the vertical position for the first time?

Solution:

(i) The moment of inertia for a parallel axis through the centre of mass is given by

$$\Theta_{cm} = \frac{m}{V}\int \vec{r}^2 \mathrm{d}V = \frac{m}{L}\int_{-L/2}^{L/2} x^2 \mathrm{d}x = \frac{m}{L}\frac{1}{3}\left(\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right) = \frac{1}{12}mL^2.$$

With the parallel axis theorem the moment inertia for the given axis can be obtained to

$$\underline{\underline{\Theta}} = \Theta_{cm} + m\left(\frac{L}{4}\right)^2 = \underline{\underline{\frac{7}{48}mL^2}}\ .$$

(ii) E.C.: $mg(\frac{L}{4} - \frac{L}{4}\cos\phi) = \frac{1}{2}\Theta\omega^2 \quad \Rightarrow \quad \underline{\underline{\omega}} = \sqrt{\frac{mgL(1-\cos\phi)}{2\Theta}} = \underline{\underline{\sqrt{\frac{24g(1-\cos\phi)}{7L}}}}$

(iii) E.C.: $\frac{1}{2}\Theta\omega^2 = \frac{1}{2}Ds^2 \quad \Rightarrow \quad \underline{\underline{s}} = \omega\sqrt{\frac{\Theta}{D}} = \underline{\underline{\omega\sqrt{\frac{7mL^2}{48D}}}}$

(iv) $\Theta\dot{\omega} = M = \frac{L}{4}F \quad \Rightarrow \quad \Theta\ddot{\phi} = -\frac{L}{4}mg\sin\phi \approx -\frac{L}{4}mg\phi$

$\Rightarrow \quad \underline{\underline{\ddot{\phi} + \frac{mgL}{4\Theta}\phi = 0}} \quad \Rightarrow \quad \underline{\underline{\phi(t) = \phi_0\cos(\omega t)}}$ with $\underline{\underline{\omega}} = \sqrt{\frac{mgL}{4\Theta}} = \underline{\underline{\sqrt{\frac{12g}{7L}}}}$

(v) $\underline{\underline{t_v}} = \frac{T}{4} = \frac{\pi}{2\omega} = \sqrt{\frac{\pi^2\Theta}{mgL}} = \underline{\underline{\sqrt{\frac{7\pi^2 L}{48g}}}}$

3. Exercise

A mass m_1 is performing an circular motion with period T and an radius R. At $t = 0\,\mathrm{s}$ this mass will hit a mass m_2 at rest in a total inelastic collision.

(i) What would be the angular velocity of the mass m_1 before the collision?

(ii) Find the velocity of the mass m_1 before the collision.

(iii) Determine the velocity u of both masses after the collision.

(iv) How much energy Q is lost in the collision?

(v) Find the new radius R' if the circular motion is caused by a magnetic field B and the total mass carries a charge q.

(vi) What would be the new period T'?

Solution:

(i) $\underline{\underline{\omega = \frac{2\pi}{T}}}$

(ii) $\underline{\underline{v}} = R\omega = \underline{\underline{\frac{2\pi R}{T}}}$

(iii) $\underline{\underline{u}} = \frac{m_1}{m_1+m_2} v = \underline{\underline{\frac{2\pi R m_1}{T(m_1+m_2)}}}$

(iv) $\underline{\underline{Q}} = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2} v^2 = \underline{\underline{\frac{2\pi^2 R^2}{T^2}\frac{m_1 m_2}{m_1+m_2}}}$

(v) $quB = \frac{(m_1+m_2)u^2}{R'} \quad \Rightarrow \quad \underline{\underline{R'}} = \frac{(m_1+m_2)u}{qB} = \underline{\underline{\frac{2\pi R m_1}{qBT}}}$

(vi) $\underline{\underline{T'}} = \frac{2\pi}{\omega'} = \frac{2\pi R'}{u} = \underline{\underline{\frac{2\pi(m_1+m_2)}{qB}}}$

4. Exercise

An electron with charge e and mass m_e enters with velocity v in x-direction an area where an homogeneous electric field E in y-direction is present.

(i) What voltage has to be applied to a parallel plate capacitor with area $A = \ell^2$, distance d to produce such an electric field? How much charges are needed on the plates?

(ii) Find the acceleration of the electron?

(iii) How long does it take to travel a distance s in x-direction?

(iv) How far and in which direction would it be deflected from its original moving direction?

(v) What would be then the absolute value of its velocity?

(vi) Calculate these quantities for $\ell = 1\,\mathrm{km}$, $d = 2\,\mathrm{m}$, $v = 2 \cdot 10^6\,\frac{\mathrm{m}}{\mathrm{s}}$, $E = 400\,\frac{\mathrm{N}}{\mathrm{C}}$, $e = 1.6 \cdot 10^{-19}\,\mathrm{C}$, $m_e = 9.1 \cdot 10^{-31}\,\mathrm{kg}$ and $s = 10\,\mathrm{cm}$ ($\epsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$).

Solution:

(i) $E = \frac{U}{d} \quad \Rightarrow \quad \underline{\underline{U = Ed}}$

$\frac{Q}{U} = C = \epsilon_0 \frac{\ell^2}{d} \quad \Rightarrow \quad \underline{\underline{Q}} = \epsilon_0 \frac{\ell^2}{d} U = \underline{\underline{\epsilon_0 \ell^2 E}}$

(ii) $F = eE \quad \Rightarrow \quad \underline{\underline{a = \frac{eE}{m_e}}}$

(iii) $v = \frac{s}{t_s} \quad \Rightarrow \quad \underline{\underline{t_s = \frac{s}{v}}}$

(iv) $\underline{\underline{\Delta y}} = \frac{1}{2}at^2 = \underline{\underline{\frac{eEs^2}{2m_e v^2}}}$

$\tan\alpha = \frac{at}{v} \quad \Rightarrow \quad \underline{\underline{\alpha = \arctan\frac{eEs}{m_e v^2}}}$

(v) $\underline{\underline{v_{tot}}} = \sqrt{v^2 + (at)^2} = \underline{\underline{v\sqrt{1 + \frac{e^2E^2s^2}{m_e^2 v^4}}}}$

(vi) $\underline{\underline{U = 800\,\mathrm{V}}}$ $\quad \underline{\underline{Q = 3.5\,\mathrm{mC}}}$ $\quad \underline{\underline{a = 7.0 \cdot 10^{13}\,\frac{\mathrm{m}}{\mathrm{s}^2}}}$

$\underline{\underline{t_s = 50\,\mathrm{ns}}}$ $\quad \underline{\underline{\Delta y = 88\,\mathrm{mm}}}$ $\quad \underline{\underline{\alpha = 60^\circ = 1.1\,\mathrm{rad}}}$

$\underline{\underline{v_{tot} = 4.0 \cdot 10^6\,\frac{\mathrm{m}}{\mathrm{s}}}}$

5. Exercise

A rod with mass m is moving with velocity v on two rails towards an inclined plane. The angle of the inclined plane with respet to the horizontal amounts to α and the rails are connected by resistance R at the bottom of the inclined plane. The rod is exposed to gravitational force.

(i) At which height the velocity will vanish?

(ii) What distance s on the inclined plane the rod has travelled until it reaches that position?

(iii) What is the total force acting on the rod on the inclined plane?

Afterwards a magnetic field B is applied perpendicular to the plane.

(iv) How large will be the terminal current I?

(v) Determine the terminal induced voltage U_{ind}

(vi) Find the final velocity v of the rod.

(vii) State the power P_F provided by gravity.

Solution:

(i) $\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad \underline{\underline{h = \frac{v^2}{2g}}}$

(ii) $\sin\alpha = \frac{h}{s} \quad \Rightarrow \quad \underline{\underline{s}} = \frac{h}{\sin\alpha} = \underline{\underline{\frac{v^2}{2g\sin\alpha}}}$

(iii) $\underline{\underline{F_D = mg\sin\alpha}}$

(iv) $F_D = IB\ell \quad \Rightarrow \quad \underline{\underline{I}} = \frac{F_D}{B\ell} = \underline{\underline{\frac{mg\sin\alpha}{B\ell}}}$

(v) $\underline{\underline{U_{\text{ind}}}} = RI = \underline{\underline{\frac{Rmg\sin\alpha}{B\ell}}}$

(vi) $U_{\text{ind}} = \ell v B \quad \Rightarrow \quad \underline{\underline{v}} = \frac{U_{\text{ind}}}{\ell B} = \underline{\underline{\frac{Rmg\sin\alpha}{B^2\ell^2}}}$

(vii) $\underline{\underline{P_F}} = vF_D = \underline{\underline{\frac{Rm^2g^2\sin^2\alpha}{B^2\ell^2}}}$

6. Exercise

An empty coil with length ℓ, turns N, area A and inner resistance r is connected to two parallel resistors R_1 and R_2. Initially a magnetic field flux Φ_0 is running through the coil, but is switched of at $t = 0\,\text{s}$.

(i) What is the maximal current I_0 which is running through the coil?

(ii) Determine the equivalent resistance of the whole circuit.

(iii) Derive the power $P_{r,0}$ which is dissipated by the internal resistance of the coil at $t = 0\,\text{s}$?

(iv) Find the power $P_{p,0}$ which is provided by the coil at $t = 0\,\mathrm{s}$.

(v) How large is the maximal voltage $U_{R,0}$ that drops at the two resistors?

(vi) State the maximal currents $I_{1,0}$ and $I_{2,0}$ through the resistors R_1 and R_2.

(vii) Give the current $I_2(t)$ through the resistor R_2 with respect to the time.

Solution:

(i) $B = \mu_0 \frac{N}{\ell} I_0,\ \Phi_0 = BAN \qquad \Rightarrow \qquad \underline{\underline{I_0}} = \frac{B\ell}{\mu_0 N} = \underline{\underline{\frac{\Phi_0 \ell}{\mu_0 A N^2}}}$

(ii) $\underline{\underline{R_{eq} = r + \frac{R_1 R_2}{R_1 + R_2}}}$

(iii) $\underline{\underline{P_{p,0}}} = (U_0 - U_{r,0}) I_0 = (R_{eq} - r) I_0^2 = \underline{\underline{\frac{R_1 R_2}{R_1 + R_2} \frac{\Phi_0^2 \ell^2}{\mu_0^2 A^2 N^4}}}$

(iv) $\underline{\underline{P_{r,0}}} = U_{r,0} I_0 = r I_0^2 = \underline{\underline{\frac{r \Phi_0^2 \ell^2}{\mu_0^2 A^2 N^4}}}$

(v) $\underline{\underline{U_{R,0}}} = (U_0 - U_{r,0}) = (R_{eq} - r) I_0 = \underline{\underline{\frac{R_1 R_2}{R_1 + R_2} \frac{\Phi_0 \ell}{\mu_0 A N^2}}}$

(vi) $\underline{\underline{I_{1,0}}} = \frac{U_{R,0}}{R_1} = \underline{\underline{\frac{R_2}{R_1 + R_2} \frac{\Phi_0 \ell}{\mu_0 A N^2}}}$

$\underline{\underline{I_{2,0}}} = \frac{U_{R,0}}{R_2} = \underline{\underline{\frac{R_1}{R_1 + R_2} \frac{\Phi_0 \ell}{\mu_0 A N^2}}}$

(vii) $\underline{\underline{I_2(t)}} = I_{2,0} \mathrm{e}^{-\frac{R_{eq}}{L} t} = \underline{\underline{\frac{R_1}{R_1 + R_2} \frac{\Phi_0 \ell}{\mu_0 A N^2} \exp\left(-\frac{(R_1 + R_2) r + R_1 R_2}{(R_1 + R_2) \mu_0 A N^2} \ell t \right)}}$