Pre-Semester Physics - Exercises Summer 2010

Stefan Kremer
stefan.kremer@ensicaen.fr

Sheet 1
9.8.2010

1. Exercise:

Differentiate the following functions with respect to the arguments two times and sketch the following functions:

(a) $f(x) = -\frac{1}{2}x^2 + \frac{3}{2}x + \frac{1}{4}$

(b) $g(y) = 1 - \mathrm{e}^{-3y}$

(c) $h(z) = \sin(2z)$

(d) $j(z) = 2\sin(z)\cos(z)$

(e) $k(t) = \mathrm{e}^{-t/4}\cos(2t)$

Solution:

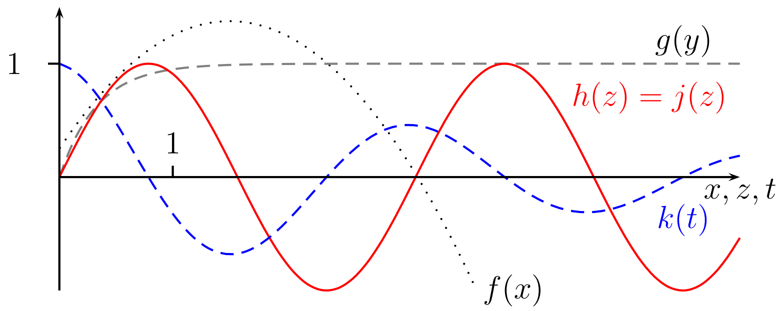
(a) $f'(x) = -x + \frac{3}{2}$,
$f''(x) = -1$

(b) $g'(y) = 3\mathrm{e}^{-3y}$,
$g''(y) = -9\mathrm{e}^{-3y}$

(c) $h'(z) = 2\cos(2z)$,
$h''(z) = -4\sin(2z)$

(d) $j'(z) = 2\cos^2(z) - 2\sin^2(z)$,
$j''(z) = -8\sin(x)\cos(x)$

(e) $k'(t) = -\mathrm{e}^{-t/4}\left(\frac{1}{4}\cos(2t) + 2\sin(2t)\right)$,
$k''(t) = -\mathrm{e}^{-t/4}\left(\frac{63}{16}\cos(2t) - \sin(2t)\right)$



2. Exercise:

With $\vec{u}_x = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\vec{u}_y = \begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\vec{u}_z = \begin{pmatrix}0\\0\\1\end{pmatrix}$ calculate

(a) $\vec{v}_{xy} = \vec{u}_x + \vec{u}_y$ and $\vec{v}_{xz} = \vec{u}_x + \vec{u}_z$,

(b) $|\vec{v}_{xy}|$ and $\vec{u}_{xy} = \frac{\vec{v}_{xy}}{|\vec{v}_{xy}|}$,

(c) $\vec{u}_x \cdot \vec{u}_y$ and $\vec{v}_{xy} \cdot \vec{v}_{xz}$,

(d) the angle $\phi_{x,xy}$ between $\vec{u}_x$ and $\vec{u}_{xy}$,

(e) $\vec{u}_x \times \vec{u}_y$, $\vec{u}_y \times \vec{u}_z$ and $\vec{u}_x \times \vec{u}_z$,

(f) $\vec{u}_x \times \vec{v}_{xy}$ and $\vec{u}_z \times \vec{v}_{xy}$,

(g) the representation of $\vec{v}_{xy}$, $\vec{u}_{xy}$, $\vec{v}_{xz}$, $\vec{u}_z \times \vec{v}_{xy}$ in a polar coordinate system where the first two Cartesian components are replaced by polar ones.

Solution:

(a) $\vec{v}_{xy} = \begin{pmatrix}1\\1\\0\end{pmatrix}, \qquad \vec{v}_{xz} = \begin{pmatrix}1\\0\\1\end{pmatrix}$

(b) $|\vec{v}_{xy}| = \sqrt{2}$

$\vec{u}_{xy} = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\\0\end{pmatrix}$

(c) $\vec{u}_x \cdot \vec{u}_y = 0$

$\vec{v}_{xy} \cdot \vec{v}_{xz} = 1$

(d) $\phi_{x,xy} = \frac{\pi}{4} = 45°$

(e) $\vec{u}_x \times \vec{u}_y = \vec{u}_z$

$\vec{u}_y \times \vec{u}_z = \vec{u}_x$

$\vec{u}_x \times \vec{u}_z = -\vec{u}_y$

(f) $\vec{u}_x \times \vec{v}_{xy} = \vec{u}_z$

$\vec{u}_z \times \vec{v}_{xy} = \begin{pmatrix}-1\\1\\0\end{pmatrix}$

(g) $\vec{v}_{xy} = \sqrt{2}\vec{u}_r + \frac{\pi}{4}\vec{u}_\phi$

$\vec{u}_{xy} = \vec{u}_r + \frac{\pi}{4}\vec{u}_\phi$

$\vec{v}_{xz} = \vec{u}_r + \vec{u}_z$

$\vec{u}_z \times \vec{v}_{xy} = \sqrt{2}\vec{u}_r + \frac{3\pi}{4}\vec{u}_\phi$

3. Exercise:

Sketch the following paths $\vec{r}(t) : [0,\pi] \to \mathbb{R}^2$:

(a) $\vec{r}(t) = \begin{pmatrix} t/\pi \\ t/\pi \end{pmatrix}$

(b) $\vec{r}(t) = \begin{pmatrix} t/\pi \\ (-t^2+t)/\pi \end{pmatrix}$

(c) $\vec{r}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

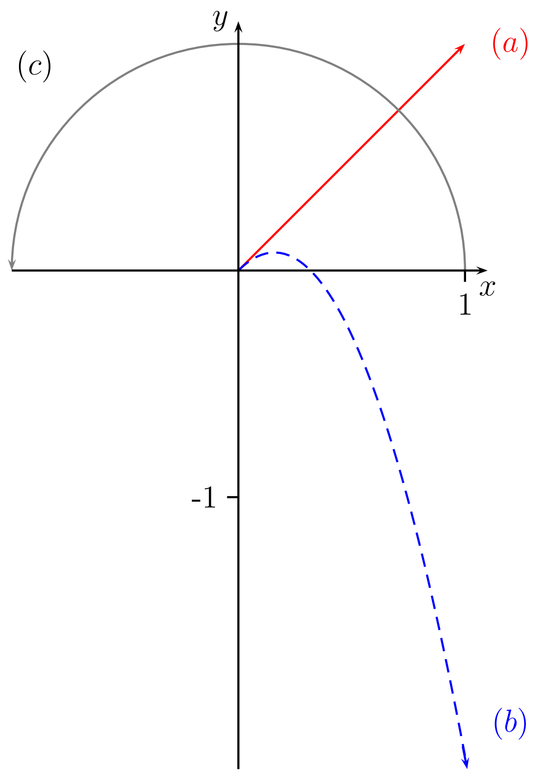
Solution:

In order to draw the path $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is convenient to eliminate t:

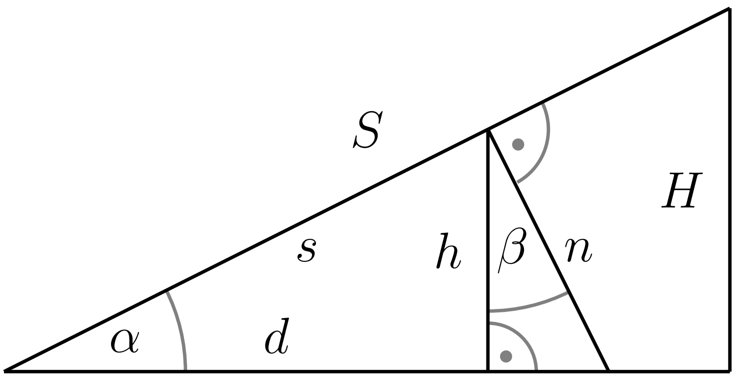
(a) $t = \pi x \quad \rightarrow \quad y = x \;$ with $\; x \in [0,1]$

(b) $t = \pi x \quad \rightarrow \quad y = -\pi x^2 + x = -\pi(x - \frac{1}{2\pi})^2 + \frac{1}{4\pi} \;$ with $\; x \in [0,1]$

(c) In a polar coordinate system the path reads $\vec{r}(t) = \vec{u}_r + t\vec{u}_\phi$. Therefore it describes half a circle: $r(\phi) = 1, \; \phi \in [0,\pi]$



4. Exercise:



Determine in the figure above

(a) $H(S, s, h)$,

(b) $H(S, \alpha)$,

(c) $h(d, \alpha)$,

(d) $\beta(\alpha)$,

(e) $d(s, \beta)$,

(f) $n(h, \alpha)$.

Solution:

(a) Since both triangles are similar: $\underline{\underline{H(S, s, h) = \frac{S}{s} h}}$,

(b) $\sin(\alpha) = \frac{H}{S} \Rightarrow \underline{\underline{H(S, \alpha) = S \sin(\alpha)}}$,

(c) $\tan(\alpha) = \frac{h}{d} \Rightarrow \underline{\underline{h(d, \alpha) = d \tan(\alpha)}}$,

(d) It holds $\pi = \frac{\pi}{2} + \beta + \left(\frac{\pi}{2} - \alpha\right) \Rightarrow \underline{\underline{\beta(\alpha) = \alpha}}$,

(e) $\cos(\alpha) = \frac{d}{s} \Rightarrow \underline{\underline{d(s, \beta) = s \cos(\beta)}}$,

(f) $\cos(\alpha) = \frac{h}{n} \Rightarrow \underline{\underline{n(h, \alpha) = \frac{h}{\cos(\alpha)}}}$.

5. Exercise:

State the following numbers with a more convenient SI-prefix and in scientific notation:

(a) $0.032\,\mathrm{m}$

(b) $2000\,\mathrm{m}$

(c) $1000000\,\mathrm{m}^2$

Solution:

(a) $0.032\,\mathrm{m} = 32\,\mathrm{cm} = 3.2\,\mathrm{mm} = 3.2 \cdot 10^{-3}\,\mathrm{m}$

(b) $2000\,\mathrm{m} = 2.000\,\mathrm{km} = 2.000 \cdot 10^{3}\,\mathrm{m}$

(c) $1000000\,\mathrm{m}^2 = 1.000000\,\mathrm{km}^2 = 1.000000 \cdot 10^{6}\,\mathrm{m}^2$

6. Exercise:

An engineer is asked to build a pipe. The instructor told him that the circumference used for the winding is 9 cm long. After a short consideration he decides to take a slip of 1 mm and build a pipe with a diameter (including the other winding) of 2.8 cm. However, when the final machine is put together the pipe does not fit. What has he done wrong?

Solution:

He has used a number with insufficient many significant digits. Therefore he should have taken a larger slip or asked for a more precise number. For instance the circumference could be $\ell = 8.5\,\mathrm{cm}$ and have been rounded to the value above. Then the real diameter needed would be $d = 2r = 2\frac{\ell}{2\pi} = \frac{\ell}{\pi} = 2.7\,\mathrm{cm}$. Thus the slip taken above was not enough. A better estimate is to use a slip of at least the half of the unit number with the minimal significant digits given, e.g. $\Delta d = 0.5\,\mathrm{cm}$.

7. Exercise:

A pizza for two persons has a certain diameter d. What diameter is needed, if four persons want to share the pizza (and get the same size as before)?

Solution:

The area every person wants to have is $A = \frac{1}{8}\pi d^2$ since the radius of the pizza is $r = \frac{d}{2}$. Thus the new pizza should have an area of $\tilde{A} = \frac{1}{4}\pi\tilde{d}^2 = 4A$. Therefore a diameter of $\underline{\underline{\tilde{d} = \sqrt{2}d}}$ is needed.

Exercises and solutions on the web:
www.tkm.uni-karlsruhe.de/∼kremer/ph10.html