KIT - International Department GmbH

Pre-Semester Physics - Exercises   Summer 2010

Stefan Kremer
stefan.kremer@ensicaen.fr

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1. **Exercise:**
   A coil with self inductance $\mathcal{L} = 5.0\,\mathrm{H}$ is placed across the terminals of a battery labeled $U = 12\,\mathrm{V}$ with inner resistance $R = 15\,\Omega$.

   (a) What is the final current I_f?

   (b) State the time constant τ.

   (c) When has the current reached 75% of its maximum value?

   (d) How large would be the maximum current $\hat{I}$ if the battery was replaced by an AC-source with frequency $\omega = 50\,\mathrm{s}^{-1}$ and maximum strength $\hat{U} = U$?

   Solution:

   (a) At the final situation the coil does not contribute. Therefore the current is given by Ohm's law:

   $$\underline{\underline{I_f}} = \frac{U}{R} = \underline{\underline{0.8\,\mathrm{A}}}$$

   (b) The time constant of this $R\mathcal{L}$-circuit is given by

   $$\underline{\underline{\tau}} = \frac{\mathcal{L}}{R} = \underline{\underline{0.33\,\mathrm{s}}}\,.$$

   (c) After the battery is connected to the coil, the current will increase exponentially:

   $$0.75 I_f = I_f \left(1 - \mathrm{e}^{-t/\tau}\right) \qquad \Rightarrow \qquad \underline{\underline{t}} = \tau \ln(4) = \underline{\underline{463\,\mathrm{ms}}}$$

   (d) The maximum current can be determined by the inductive reactance (since the inner resistance of the battery is replaced as well):

   $$\underline{\underline{\hat{I}}} = \frac{\hat{U}}{X_L} = \frac{\hat{U}}{\omega\,\mathcal{L}} = \underline{\underline{48\,\mathrm{mA}}}$$

2. Exercise:

A capacitor with $C = 4.0\,\mu\text{F}$ is charged to $U = 24\,\text{V}$ and then connected across two serially connected resistors with $R = 100\,\Omega$.

(a) Find the initial charge Q_0 on the capacitor.

(b) Determine the initial current I_0 through the resistor.

(c) What is the time constant τ?

(d) How much charge Q is on the capacitor after $t = 4\,\mu\text{s}$?

(e) Sketch the time dependence of the current $I_1(t)$ through one of the resistors and mark where I_0 and τ can be found in the diagram.

Solution:

(a) The initial charge is determined by the definition of the capacitance:

$$\underline{\underline{Q_0}} = CU = \underline{\underline{96\,\mu\text{C}}}$$

(b) The initial current is given by Ohm's law:

$$\underline{\underline{I_0}} = \frac{U}{2R} = \underline{\underline{0.12\,\text{A}}}$$

(c) The time constant for a RC-circuit where the total resistance is $2R$ was found to be

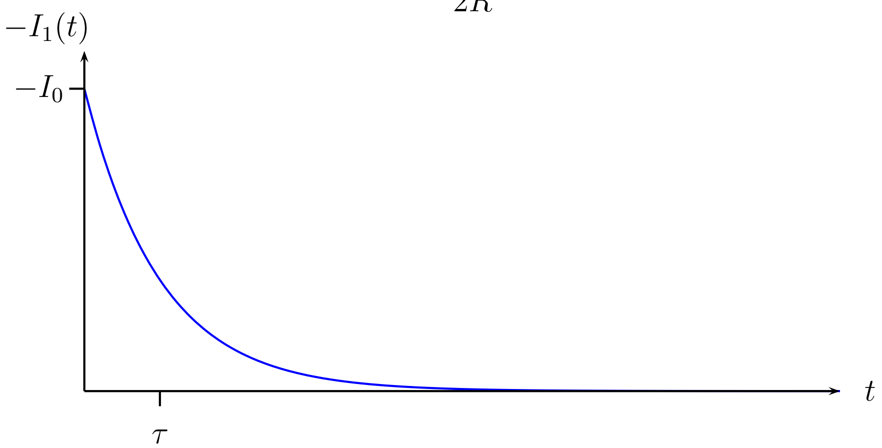
$$\underline{\underline{\tau}} = 2RC = \underline{\underline{0.8\,\text{ms}}}\ .$$

(d) The charge is exponentially decreasing:

$$\underline{\underline{Q(t)}} = Q_0 e^{-t/\tau} = \underline{\underline{95.5\,\mu\text{C}}}$$

(e) The current is given by the time derivative of the charge:

$$I_1(t) = -\frac{U}{2R} e^{-t/(2RC)} = -I_0 e^{-t/\tau}$$



- Note that in this exercise both times the total resistance 2R is present in the denominator as well as in the exponent in the current. However, in a parallel connection the current would divide into two currents flowing through the resistors and therefore different values would be present in general.

3. Exercise:

A capacitor with $C = 2.0\,\mu$F is charged to $U = 20$ V and is then connected across an inductor with $\mathcal{L} = 6.0\,\mu$H.

(a) What is the frequency f of the oscillation?

(b) Find the maximum value of the charge on the capacitor Q_0.

(c) Determine the maximum value of the current I_0.

Solution:

(a) For this situation Kirchhoff's voltage law reads

$$0 = \frac{Q}{C} + \mathcal{L}\ddot{Q} \qquad \Rightarrow \qquad \ddot{Q} = -\frac{1}{\mathcal{L}C}Q$$

which is the differential equation of the harmonic oscilator. Therefore the solution is given by

$$Q(t) = Q_0 \cos(\omega t) \qquad \text{with} \qquad \omega = \frac{1}{\sqrt{\mathcal{L}C}}$$

where the initial charge Q_0 which is initially at the capacitor was used. Hence, the frequency is given by

$$\underline{\underline{f}} = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{\mathcal{L}C}} = \underline{\underline{4.6 \cdot 10^4\,\text{Hz}}} \,.$$

(b) The maximum charge which is the initial charge is determined by the definition of the capacitance:

$$\underline{\underline{Q_0}} = CU = \underline{\underline{40\,\mu\text{C}}}$$

(c) The maximum current is determined by the maximum of the derivative of the charge:

$$\underline{\underline{I_0}} = \max_t(\dot{Q}) = \max_t(-Q_0\omega\sin(\omega t)) = \omega Q_0 = \omega C U = \underline{\underline{11.5\,\text{A}}}$$

This is like the current of the coil as an AC-source using the capacitive reactance, $I_0 = \frac{U}{X_C}$, only that the frequency of the AC-source depend as well on the capacitance. It can as well be viewed as the capacitor as an AC-source in an inductive circuit. The current is therefore given by the inductive reactance $I_0 = \frac{U}{X_L}$ where the frequency of the source depend as well on the given inductance.