Pre-Semester Physics - Exercises Summer 2010

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Sheet 2
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1. Exercise:

A person stands on a cliff which has a height of $h_1 = 80\,\mathrm{m}$ measured from the water surface. The sea itself has a depth of $h_2 = 16\,\mathrm{m}$. The person drops a stone down the cliff, which is accelerated (due to gravity) by $g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Assume the stone entering the water to propagate with the constant velocity from the point on when it is at the water surface.

(a) At what speed v_i the stone impacts with the ground of the sea?

(b) Find the time T the stone needs to hit the ground of the sea.

(c) Sketch the $x(t)$ diagram.

(d) Draw the $v(t)$ diagram.

(e) Now conclude what height $\tilde{h}_1$ the cliff has, if the person hears the sound of the impact with the sea $\tilde{t}_{ac} = 6.6\,\mathrm{s}$ after he had dropped the stone. Note that the acoustic velocity is $v_{ac} = 340\,\frac{\mathrm{m}}{\mathrm{s}}$.

Solution:

(a) Ansatz: Linear motion with constant acceleration before the stone hits the sea:

$$x_1(t) = -\frac{g}{2}t^2 + v_{1,0}t + x_{1,0}$$

After the impact on the water the stone performs a uniform motion:

$$x_2(t) = v_{20}t + x_{2,0}$$

In taking a point of origin at sea level all parameters except one are given by

$$v_{1,0} = 0, \qquad x_{1,0} = h_1, \qquad x_{2,0} = 0$$

where the time of the first motion is measured form the time when the person releases the stone while the one of the uniform motion is measured from the time the stone hits the sea. This time t_i is determined by

$$0 = x_1(t_i) = -\frac{g}{2}t_i^2 + h_1 \qquad \Rightarrow \qquad t_i = \sqrt{\frac{2h_1}{g}}\,.$$

The velocity is then

$$v_1(t_s) = -gt_i = -\sqrt{2gh_1}$$

which coincides with the missing parameter of the uniform motion $v_{20} = v_1(t_i)$ if the time of the uniform motion is measured form the time of impact t_i. Thus the speed at the ground of the sea is given by

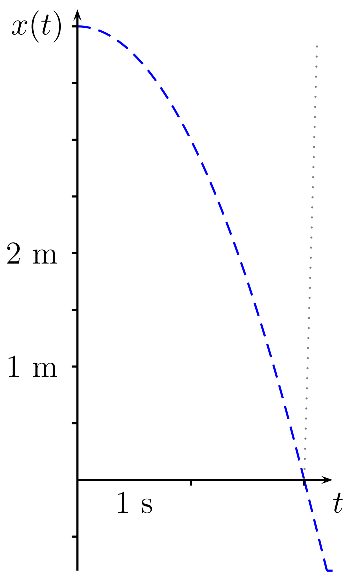
$$\underline{\underline{v_i}} = v_{20} = v_1(t_i) = -\sqrt{2gh_1} = \underline{\underline{-40\,\frac{\text{m}}{\text{s}}}}\ .$$

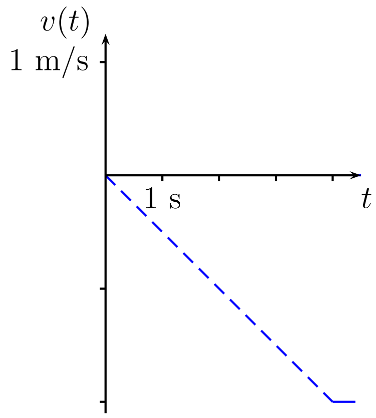
(b) The stone hits the ground of the sea at

$$\begin{aligned} -h_2 &= x_2(T - t_i) = v_{20}(T - t_i) \\ \Rightarrow \qquad \underline{\underline{T}} &= t_i - \frac{h_2}{v_{20}} = \sqrt{\frac{2h_1}{g}} + \frac{h_2}{\sqrt{2gh_1}} = 4.0\,\text{s} + 0.4\,\text{s} = \underline{\underline{4.4\,\text{s}}}\ . \end{aligned}$$

(c),(d) Since $x(t) = \begin{cases} x_1(t) & 0 \le t < t_i \\ x_2(t) & t_i \le t < T \end{cases} = \begin{cases} -5\,\frac{\text{m}}{\text{s}^2} \cdot t^2 + 80\,\text{m} & 0 \le t < 4\,\text{s} \\ -40\,\frac{\text{m}}{\text{s}} \cdot (t - 4\,\text{s}) & 4\,\text{s} \le t < 4.4\,\text{s} \end{cases}$

and $\quad v(t) = \begin{cases} \dot{x}_1(t) & 0 \le t < t_i \\ \dot{x}_2(t) & t_i \le t < T \end{cases} = \begin{cases} -10\,\frac{\text{m}}{\text{s}^2} \cdot t & 0 \le t < 4\,\text{s} \\ -40\,\frac{\text{m}}{\text{s}} & 4\,\text{s} \le t < 4.4\,\text{s} \end{cases}$





(e) After the impact the sound performs an uniform motion and therefore needs

$$\begin{aligned} x_{ac}(t) = v_{ac}(t - \tilde{t}_i) \qquad &\text{with} \qquad x_{ac}(t_{ac}) \stackrel{!}{=} \tilde{h}_1 \\ \Rightarrow \qquad t_{ac} = \tilde{t}_i + \tfrac{\tilde{h}_1}{v_{ac}} &= \sqrt{\tfrac{2\tilde{h}_1}{g}} + \tfrac{\tilde{h}_1}{v_{ac}} \end{aligned}$$

to reach the person. That means the height would now be given by

$$\begin{aligned} & v_{ac}t_{ac} - \tilde{h}_1 = v_{ac}\sqrt{\tfrac{2\tilde{h}_1}{g}} \\ \Rightarrow \quad & (v_{ac}t_{ac} - \tilde{h}_1)^2 = {v_{ac}}^2\,\tfrac{2\tilde{h}_1}{g} \\ \Rightarrow \quad & v_{ac}^2 {t_{ac}}^2 - 2(t_{ac} + \tfrac{v_{ac}}{g})v_{ac}\tilde{h}_1 + {\tilde{h}_1}^2 = 0 \\ \Rightarrow \quad & \underline{\underline{\tilde{h}_1}} = v_{ac}\left(t_{ac} + \tfrac{v_{ac}}{g} - \sqrt{\left(t_{ac} + \tfrac{v_{ac}}{g}\right)^2 - t_{ac}^2}\right) = \underline{\underline{184\,\text{m}}}\ . \end{aligned}$$

2. Exercise:

A car is racing along the street at a speed of $v = 72\,\frac{\text{km}}{\text{h}}$. At some place there is a police car spotting racers. As the racer passes the police car they start to catch up with the car driver. The police car accelerates with $a = 10\,\frac{\text{m}}{\text{s}^2}$.

(a) Find the time T the police car needs to catch up with the car driver!

(b) How far did the racer come?

(c) Draw the $x(t)$ dependencies for the racer and the police car in one diagram.

Solution:

(a) While the racer performs an uniform motion with

$$x_r(t) = vt$$

the police car performs a linear motion with constant acceleration

$$x_p(t) = \frac{1}{2}at^2\,.$$

The constant where already adjusted to the given problem. The time when both cars are at the same position (despite the one at the origin) is given by

$$vT = x_r(T) = x_p(T) = \frac{1}{2}aT^2$$

$$\Rightarrow \underline{\underline{T}} = 2\frac{v}{a} = \underline{\underline{4\,\text{s}}}\,.$$

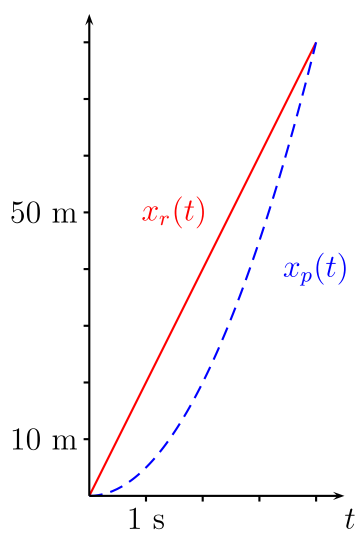
Note that the units of the velocity has been recalculated to

$$v = 72\,\frac{\text{km}}{\text{h}} = 72\,\frac{10^3\text{m}}{60\cdot 60\cdot \text{s}} = \frac{72}{3.6}\,\frac{\text{m}}{\text{s}} = 20\,\frac{\text{m}}{\text{s}}\,.$$

(b) At this time the racer will be at

$$\underline{\underline{x_r(T)}} = vT = \underline{\underline{80\,\text{m}}}\,.$$

(c)



3. Exercise:

A mass point is shot with an initial velocity of $v_0 = 20\,\frac{\text{m}}{\text{s}}$ at $\alpha = 45°$ from the horizontal. Along the vertical gravity is acting with $g = 10\,\frac{\text{m}}{\text{s}^2}$.

(a) Sketch the path $\vec{r}(t)$.

(b) What is the total time T the mass point is in the air?

(c) Find the range, the total horizontal distance s traveled.

(d) Determine the maximum height h of the mass point.

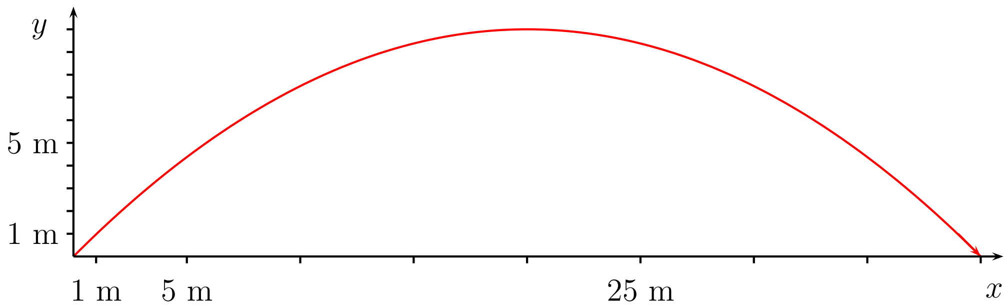
(e) How would the range s change if the angle would be increased?

Solution:

(a) Along the x-direction a uniform motion is taking place, while along the y direction a motion with constant acceleration is been performed:

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} v_0 \cos(\alpha) \cdot t \\ -\frac{1}{2}gt^2 + v_0 \sin(\alpha) \cdot t \end{pmatrix} = \begin{pmatrix} 14\,\frac{\text{m}}{\text{s}}t \\ -5\,\frac{\text{m}}{\text{s}^2}t^2 + 14\,\frac{\text{m}}{\text{s}}t \end{pmatrix}$$

$$\Rightarrow \quad t = \frac{x}{v_0 \cos(\alpha)}, \qquad y = -\frac{g}{2{v_0}^2 \cos^2(\alpha)}x^2 + \tan(\alpha) \cdot x = -0.025\,\text{m}^{-1}x^2 + x$$



(b) Since the time is ask when the y-coordinate will again vanish it is given by

$$y(t) = -\tfrac{1}{2}gt^2 + v_0 \sin(\alpha) \cdot t \qquad \text{with} \qquad y(T) \overset{!}{=} 0$$

$$\Rightarrow \qquad \underline{\underline{T}} = \tfrac{2v_0}{g} \sin(\alpha) = \underline{\underline{2.8\,\text{s}}}\ .$$

(c) The range is given by the value of the x-coordinate at this time:

$$\underline{\underline{s}} = x(T) = v_0 \cos(\alpha)T = \frac{v_0^2}{g} \sin(2\alpha) = \underline{\underline{40\,\text{m}}}$$

(d) At the maximum point the y-component of the velocity vanishes. Therefore the mass point would be at its maximum height at

$$v_y(t) = -gt + v_0 \sin(\alpha) \qquad \text{with} \qquad v_y(t) \stackrel{!}{=} 0$$
$$\Rightarrow \qquad \tilde{T} = \tfrac{v_0}{g} \sin(\alpha) = \tfrac{T}{2}$$

as can also be argued by symmetry. The height is then given by the y-coordinate:

$$\underline{\underline{h}} = y(\tilde{T}) = -\frac{1}{2} g \tilde{T}^2 + v_0 \sin(\alpha) \tilde{T} = \frac{{v_0}^2}{2g} \sin^2(\alpha) = \underline{\underline{10\,\mathrm{m}}}$$

(e) Since the range depends on the angle only via $\sin(2\alpha)$ the range will <u>decrease</u> for any other angle.