Pre-Semester Physics - Exercises   Summer 2010

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1. Exercise:

The participants of the "Tour de France" ride on their bikes at an average speed of $v = 45\,\frac{\text{km}}{\text{h}}$. Suppose the stage has a distance of $s = 300\,\text{km}$. The wheels have a diameter of $d = 70\,\text{cm}$.

(a) When does the bikes reach the finish?

(b) How often do the wheels turn around?

(c) What is the time of circulation T of a reflector on the wheel?

(d) Find the angular velocity ω and the frequency f of the wheels!

(e) Determine the absolute value of the acceleration a the spokes of the wheel has to withstand.

Solution:

(a) From an assumed uniform motion the time the bikes reach the finish line is given by

$$v = \frac{s}{t_f} \qquad \Rightarrow \qquad \underline{\underline{t_f}} = \frac{s}{v} = 24\,000\,\text{s} \qquad \text{or} \qquad \underline{\underline{6\,\text{h}\ 40\,\text{min}}}\ .$$

Note that the units of the velocity has to be recalculated to

$$v = 45\,\frac{\text{km}}{\text{h}} = 45\,\frac{10^3\text{m}}{60 \cdot 60 \cdot \text{s}} = \frac{45}{3.6}\,\frac{\text{m}}{\text{s}} = 12.5\,\frac{\text{m}}{\text{s}}\,.$$

(b) During one turn of the wheel the bikes will move along the distance of the circumference $\tilde{s} = \pi d$ of the wheel, therefore the total number of turns $\tilde{n}$ can be calculated by

$$\pi d n \stackrel{!}{=} s \qquad \Rightarrow \qquad \underline{\underline{\tilde{n}}} = \lfloor n \rfloor = \left\lfloor \frac{s}{\pi d} \right\rfloor = \underline{\underline{136\,418}}\ .$$

(c) Since they turn around n many times during the time span t_f the time of circulation is determined by

$$\underline{\underline{T}} = \frac{t_f}{n} = \underline{\underline{0.176\text{s}}}\ .$$

(d) From it the angular velocity can be determined:

$$\underline{\underline{\omega = \frac{2\pi}{T} = 36\,\mathrm{s}^{-1}}} \qquad \underline{\underline{f = \frac{1}{T} = 5.7\,\mathrm{s}^{-1}}}$$

which coincides with the one directly calculated by the motion

$$v = \omega r = \omega \frac{d}{2} \qquad \Rightarrow \qquad \omega = \frac{2v}{d} = 36\,\mathrm{s}^{-1}\,.$$

(e) The spokes has to withstand the centrifugal force given through the centrifugal acceleration

$$\underline{\underline{a_c}} = \omega^2 \frac{d}{2} = \underline{\underline{446\,\frac{\mathrm{m}}{\mathrm{s}^2}}}\,.$$

2. Exercise:

Consider two persons standing on a whirligig, a rotating disc with radius r, at constant angular velocity ω. One person is standing at the middle point, the other one at the border. The one at the border likes to throw a ball to the person in the middle with some certain velocity v.

(a) What is the velocity v_b of the person at the border?

(b) At what angle α in the plane of the disc and with respect to the line joining the persons does the person have to throw the ball.

(c) How long will it take until the ball reaches the person in the middle?

(d) Now suppose the person can only throw the ball with a maximum velocity $v_{\max}$. What is the maximal angular velocity $\omega_{\max}$ allowing the person at the border to reach the person in the middle with the ball?

(e) Consider the person in the middle who wants to throw back the ball to the person outside. In which direction does he have to throw the ball and how long will it take?

Solution:

(a) The speed at the border of the disc is given by

$$\underline{\underline{v_b = r\omega}}\,.$$

(b) Since the ball has already a velocity v_b perpendicular to the desired throwing direction at the beginning the angle is determined by

$$\sin(\alpha) = \frac{v_b}{v} = \frac{\omega r}{v} \qquad \Rightarrow \qquad \underline{\underline{\alpha = \arcsin\left(\frac{\omega r}{v}\right)}}\,.$$

(c) The ball travels along the radial coordinate with the velocity

$$v_{\mathrm{r}} = v^2 - v_b^2 \,.$$

Therefore it has passed the radius after

$$r = v_{\mathrm{r}} t \qquad \Rightarrow \qquad \underline{\underline{t = \frac{r}{v_{\mathrm{r}}} = \frac{r}{\sqrt{v^2 - r^2\omega^2}}}} \,.$$

(d) Mathematically the sinus in (b) can not be inverted if

$$\omega > \underline{\underline{\omega_{\mathrm{max}} = \frac{v_{\mathrm{max}}}{r}}} \,.$$

(e) The throw will longer since the ball will be at rest at the beginning:

$$r = vt \qquad \Rightarrow \qquad \underline{\underline{t = \frac{r}{v}}}$$

The person should aim at the position of the other person after this time which has moved along the circle the distance $r\alpha$ (α in rad!) where the angle is given by the angular coordinate $r_\phi(t) = \omega t$:

$$r\alpha = r\omega t \qquad \Rightarrow \qquad \underline{\underline{\alpha = \frac{\omega r}{v}}}$$