KIT - International Department GmbH

Pre-Semester Physics - Exercises   Summer 2010

Stefan Kremer
stefan.kremer@ensicaen.fr

Sheet 4
18.8.2010

1. Exercise:

An object with mass $m = 27\,\mathrm{kg}$ is pulled along a frictionless horizontal surface by a horizontal force $F = 24\,\mathrm{N}$.

(a) If the object is at rest at $t = 0$ how fast is it at $t = 3.0\,\mathrm{s}$?

(b) How far does it travel during this time?

(c) What distance s would be between its position at $t = 3.0\,\mathrm{s}$ and its original position if it had an initial velocity of $v_0 = 1.0\,\frac{\mathrm{m}}{\mathrm{s}}$ perpendicular to the force?

Solution:

(a) Its acceleration is given by Newton's second axiom:

$$F = ma \qquad \Rightarrow \qquad a = \frac{F}{m}$$

Since it is constant and at rest at the beginning, it performs a linear motion with constant acceleration. Therefore the velocity after the time t is given by

$$v(t) = at = \frac{F}{m}t \qquad \Rightarrow \qquad \underline{\underline{v(3.0\,\mathrm{s}) = 2.7\,\frac{\mathrm{m}}{\mathrm{s}}}}\ .$$

(b) The position for this kind of motion is determined by

$$x(t) = \frac{1}{2}at^2 = \frac{F}{2m}t^2 \qquad \Rightarrow \qquad \underline{\underline{x(3.0\,\mathrm{s}) = 4.0\,\mathrm{m}}}\ .$$

(c) Due to the initial velocity the mass will perform an uniform motion along the direction perpendicular to the force. The position for this kind of motion can be evaluated by

$$y(t) = v_0 t\,.$$

Since this motion is perpendicular to the former one the motions along both directions will be performed independently. Therefore the distance as given by the Pythagorean theorem states

$$\underline{\underline{s}} = \sqrt{x(3.0\,\mathrm{s})^2 + y(3.0\,\mathrm{s})^2} = \sqrt{16\,\mathrm{m}^2 + 9\,\mathrm{m}^2} = \underline{\underline{5\,\mathrm{m}}}\ .$$

2. Exercise:

Two masses $m_1 = 2\,\text{kg}$ and $m_2 = 3\,\text{kg}$ are connected by a rope which runs over a pulley. Both masses are exposed to the gravitational force ($g = 10\frac{m}{s^2}$). However, the mass m_1 lies on an inclined plane with angle $\alpha = 30°$ (with respect to the horizontal) while the other one is hanging in the air, $h = 2.0\,\text{m}$ above ground level. Neglect friction!

(a) Determine the acceleration a of the system in terms of g.

(b) Find the time t, the mass m_1 needs to hit the ground.

(c) Find the tension force F_T in the rope.

(d) What ratio of m_1 and m_2 is needed, such that the system stays at rest?

Solution:

(a) Newton's second axiom for both masses reads

$$\begin{aligned} m_1 a_1 &= -m_1 g \sin\alpha + F_T \\ m_2 a_2 &= \quad m_2 g \qquad\quad - F_T \end{aligned}$$

where the forces were measured in the direction to the pulley for mass m_1 and away from the pulley for mass m_2. If the mass m_2 moves along the vertical, the mass m_1 would move due to the rope the same distance along the inclined plane. That means

$$x_1 = x_2 \qquad \Rightarrow \qquad \ddot{x}_1 = \ddot{x}_2 \qquad \text{or} \qquad a_1 = a_2 = a$$

if measured from the initial positions. Therefore we can eliminate the tension force F_T in the equations of motion:

$$\begin{aligned} F_T &= m_1 a + m_1 g \sin\alpha \\ m_2 a &= m_2 g - m_1 g \sin\alpha - m_1 a \qquad \Rightarrow \qquad \underline{a} = \frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} g = \underline{\underline{0.4g}} \end{aligned}$$

(b) Since the masses perform a linear motion with the constant acceleration determined above, the mass m_2 hits the ground after

$$h = \frac{1}{2} a t^2 \qquad \Rightarrow \qquad \underline{t} = \sqrt{\frac{2h}{a}} = \underline{\underline{1\,\text{s}}}\ .$$

(c) As determined in the first part the tension force reads

$$\underline{\underline{F_T}} = m_1 (a + g \sin\alpha) = \underline{\underline{18\,\text{N}}}\ .$$

(d) Inverting the formula for a while putting this value to zero states

$$0 = m_2 - m_1 \sin\alpha \qquad \Rightarrow \qquad \underline{\underline{\frac{m_2}{m_1} = \sin(\alpha)}}\ .$$

3. Exercise:

Consider the fall of a stone into a lake (due to gravity with $g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$): At $t = 0$ the stone with mass $m = 1.0\,\mathrm{g}$ is released at the water surface. There is friction induced by moving through the water with coefficient $\gamma = 1.0\,\frac{\mathrm{g}}{\mathrm{s}}$.

(a) What is the total force F_{tot} on the stone depending on the velocity of the stone?

(b) State the equation of motion.

(c) What would be the final constant velocity v_{max} the stone could reach?

(d) Prove that the distance traveled in the water as given by

$$x(t) = -10\,\mathrm{m} \cdot \left(1 - t \cdot \mathrm{s}^{-1} - \mathrm{e}^{-t \cdot \mathrm{s}^{-1}}\right)$$

solves the equation of motion.

Solution:

(a) Since the friction force points in the opposite direction as gravity the total force reads

$$\underline{\underline{F_{\mathrm{tot}}}} = mg - \gamma v = \underline{\underline{10\,\mathrm{mN} - 1.0\,\frac{\mathrm{g}}{\mathrm{s}} \cdot v}} \quad .$$

(b) The equation of motion as determined by Newton's second axiom states

$$\underline{\underline{m\ddot{x}}} = F_{\mathrm{tot}} = mg - \gamma v = \underline{\underline{10\,\mathrm{mN} - 1.0\,\frac{\mathrm{g}}{\mathrm{s}} \cdot \dot{\mathrm{x}}}} \quad .$$

(c) For a constant velocity the acceleration on the left-hand side of the equation of motion should vanish. Therefore the final velocity is given by

$$0 = mg - \gamma v \qquad \Rightarrow \qquad \underline{\underline{v}} = \frac{mg}{\gamma} = \underline{\underline{10\,\frac{\mathrm{m}}{\mathrm{s}}}} \quad .$$

(d) In order to show that the given function fulfills the equation of motion, the quantities appearing there are determined by the derivatives:

$$\begin{aligned} x(t) &= -10\,\mathrm{m} \cdot \left(1 - t \cdot \mathrm{s}^{-1} - \mathrm{e}^{-t \cdot \mathrm{s}^{-1}}\right) \,, \\ \dot{x}(t) &= 10\,\frac{\mathrm{m}}{\mathrm{s}} \cdot \left(\quad 1 \quad - \mathrm{e}^{-t \cdot \mathrm{s}^{-1}}\right) \,, \\ \ddot{x}(t) &= 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \quad \mathrm{e}^{-t \cdot \mathrm{s}^{-1}} \quad . \end{aligned}$$

Putting them in the left-hand and right-hand side of the equation of motion leads to the same result:

$$\begin{aligned} m\ddot{x} &= m\ddot{x}(t) = 1\,\mathrm{g} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{e}^{-t \cdot \mathrm{s}^{-1}} = 10\,\mathrm{mN} \cdot \mathrm{e}^{-t \cdot \mathrm{s}^{-1}} \\ mg - \gamma\dot{x} &= 10\,\mathrm{mN} - 1.0\,\frac{\mathrm{g}}{\mathrm{s}} \cdot \left(10\,\frac{\mathrm{m}}{\mathrm{s}} - 10\,\frac{\mathrm{m}}{\mathrm{s}} \cdot \mathrm{e}^{-t \cdot \mathrm{s}^{-1}}\right) = 10\,\mathrm{mN} \cdot \mathrm{e}^{-t \cdot \mathrm{s}^{-1}} \end{aligned}$$

Therefore the given function fulfills the equation of motion.

4. Exercise:

A pendulum consists of a mass point with $m = 2\,\mathrm{kg}$ attached to a rope of length $\ell = 3\,\mathrm{m}$. The mass point is pushed in horizontal direction with a velocity of $v = 4.5\,\frac{\mathrm{m}}{\mathrm{s}}$. Consider the situation at which the rope possesses an angle of $\alpha = 30°$ with respect to the vertical line.

(a) Determine the potential energy difference between both situations.

(b) Find the velocity v'.

(c) Which angle α_{max} does it reach at its highest point?

Solution:

(a) At a finite angle the mass is lifted by

$$h = \ell(1 - \cos\alpha)\,.$$

as given by the definition of the cosine in the triangle of the point of rotation, the new position and it's projection on the vertical. Therefore the potential energy is given by

$$\underline{\underline{E'_{pot}}} = mgh = mg\ell(1 - \cos\alpha) = \underline{\underline{8.0\,\mathrm{J}}}\,.$$

(b) The particle has only kinetic energy in the former position as given by

$$E_{\mathrm{kin}} = \frac{1}{2}mv^2\,.$$

Therefore from the conservation of energy combined with the former result the velocity can be determined:

$$\begin{aligned} E_{\mathrm{kin}} &= E'_{\mathrm{kin}} + E'_{\mathrm{pot}} \\ \frac{1}{2}mv^2 &= \frac{1}{2}mv'^2 + mg\ell(1 - \cos\alpha) \\ \Rightarrow \quad \underline{\underline{v'}} &= \sqrt{v^2 - 2g\ell(1 - \cos\alpha)} = \underline{\underline{3.5\,\frac{\mathrm{m}}{\mathrm{s}}}} \end{aligned}$$

(c) At the maximum point only potential energy is present. Therefore the conservation of energy reads

$$\begin{aligned} \frac{1}{2}mv^2 &= mgh = mg\ell(1 - \cos\alpha) \\ \Rightarrow \quad \underline{\underline{\alpha}} &= \arccos\left(1 - \frac{v^2}{2g\ell}\right) = \underline{\underline{48.5°}}\,. \end{aligned}$$

5. Exercise:

A mass $m_1 = 1\,\mathrm{kg}$ is situated on an inclined plane with angle $\alpha = 30°$. At $t = 0$ it starts moving towards a mass $m_2 = 9\,\mathrm{kg}$ at a distance of $s = 0.4\,\mathrm{m}$ to the first mass and on the same inclined plane. However, the position of the second mass is fixed until both masses collide.

(a) When does the masses collide?

(b) State the velocity v at the time of impact.

(c) What are the velocities $v_1',\ v_2'$ after the total elastic collision of the two?

(d) Find the height h the first mass reaches after the collision.

(e) Determine the contraction x of a spring with constant $D = 16\,\frac{\mathrm{N}}{\mathrm{m}}$ which the second mass reaches after the collision.

Solution:

(a) Since it performs an linear motion with constant acceleration given by $a_D = g \sin\alpha$ the time is determined to

$$\underline{t} = \sqrt{\frac{2s}{g\sin\alpha}} = \underline{\underline{0.4\,\mathrm{s}}}\ .$$

(b) Putting this result into the time dependent velocity results in

$$\underline{\underline{v}} = g\sin\alpha \cdot t = \underline{\underline{2\,\frac{\mathrm{m}}{\mathrm{s}}}}\ .$$

(c) As derived in the script, the velocities are given by

$$\underline{\underline{v_1'}} = \frac{(m_1 - m_2)v + 2m_2 \cdot 0}{m_1 + m_2} = \underline{\underline{-1.6\,\frac{\mathrm{m}}{\mathrm{s}}}}\ ,$$

$$\underline{\underline{v_2'}} = \frac{(m_2 - m_1)\cdot 0 + 2m_1 v}{m_1 + m_2} = \underline{\underline{0.4\,\frac{\mathrm{m}}{\mathrm{s}}}}\ .$$

(d) The height can be calculated by the conservation of energy:

$$\frac{1}{2} m_1 {v_1'}^2 = m_1 g h \qquad \Rightarrow \qquad \underline{\underline{h}} = \frac{{v_1'}^2}{2g} = \underline{\underline{128\,\mathrm{mm}}}$$

(e) In order to determine the contraction x the conservation of energy is used as well:

$$\frac{1}{2} m_2 {v_2'}^2 = \frac{1}{2} D x^2 \qquad \Rightarrow \qquad \underline{\underline{x}} = v_2' \sqrt{\frac{m_2}{D}} = \underline{\underline{30\,\mathrm{cm}}}$$

6. **Exercise:**

Consider again the police car catching a racer. The racer travels at a speed of $v = 72\,\frac{\text{km}}{\text{h}}$ and has a mass of $m_r = 900\,\text{kg}$ while the police car has mass $m_p = 1100\,\text{kg}$ and accelerates with $a = 10\,\frac{\text{m}}{\text{s}^2}$. However, the police car does not break in time and therefore crashes in the racer. After the crash the police car and the racer stick together.

(a) What is the speed of the cars after the crash?

(b) How much energy is lost in the collision?

(c) If the racer would be modeled by a spring with constant $D = 1000\,\frac{\text{N}}{\text{m}}$, how large would be the deformation, i.e. the contraction of the car?

Solution:

(a) Since the racer and the police car meet at $T = 4\,\text{s}$ their velocities are then

$$v_r = v = 20\,\frac{\text{m}}{\text{s}}, \qquad v_p = aT = 40\,\frac{\text{m}}{\text{s}}\,.$$

From momentum conservation the final velocity is obtained:

$$m_r v_r + m_p v_p = (m_r + m_p)v \qquad \Rightarrow \qquad \underline{\underline{v}} = \frac{m_r v_r + m_p v_p}{m_r + m_p} = \underline{\underline{31\,\frac{\text{m}}{\text{s}}}}$$

(b) The energy loss is given by energy conservation:

$$\begin{aligned} \frac{1}{2}m_r {v_r}^2 \ + \ \frac{1}{2}m_p {v_p}^2 &= \frac{1}{2}(m_r + m_p)v^2 \ + \ Q \\ \underline{\underline{Q}} \ &= \ \frac{1}{2}\frac{(m_r + m_p)m_r {v_r}^2 + (m_r + m_p)m_p {v_p}^2 - (m_r v_r + m_p v_p)^2}{m_r + m_p} \\ &= \ \frac{1}{2}\frac{m_p m_r}{m_r + m_p}(v_r - v_p)^2 = \underline{\underline{99\,\text{kJ}}} \end{aligned}$$

(c) We can picture the deformation process as the source where the energy is lost. In this way, the deformation s can be obtain by the conservation of energy:

$$Q = \frac{1}{2}Ds^2 \qquad \Rightarrow \qquad \underline{\underline{s}} = \sqrt{\frac{2Q}{D}} = \underline{\underline{14.1\,\text{m}}}$$

Therefore the crash would be lethal!

Note:

Consider $g = 10\,\frac{\text{m}}{\text{s}^2}$.