KIT - International Department GmbH

Pre-Semester Physics - Exercises Summer 2010

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1. Exercise:

A body of unknown mass is attached to an ideal spring with force constant $D = 120\,\frac{\mathrm{N}}{\mathrm{m}}$. It is found to vibrate with a frequency of $f = 0.6\,\mathrm{Hz}$. At $t = 0$ the particle is at rest at $x_0 = 10\,\mathrm{cm}$.

(a) What is the period T?

(b) Determine the mass m of the body.

(c) Find the velocity v where the spring is neither contracted nor elongated.

(d) Sketch $x(t)$.

(e) How would the diagram change if the mass would move in water?

Solution:

(a) The period is given by

$$\underline{\underline{T}} = \frac{1}{f} = \underline{\underline{1.7\,\mathrm{s}}}\ .$$

(b) The mass can be found from the frequency of the simple oscillator:

$$\omega = \sqrt{\frac{D}{m}} \qquad \Rightarrow \qquad \underline{\underline{m}} = \frac{D}{\omega^2} = \frac{D}{4\pi^2 f^2} = \underline{\underline{8.4\,\mathrm{kg}}}$$

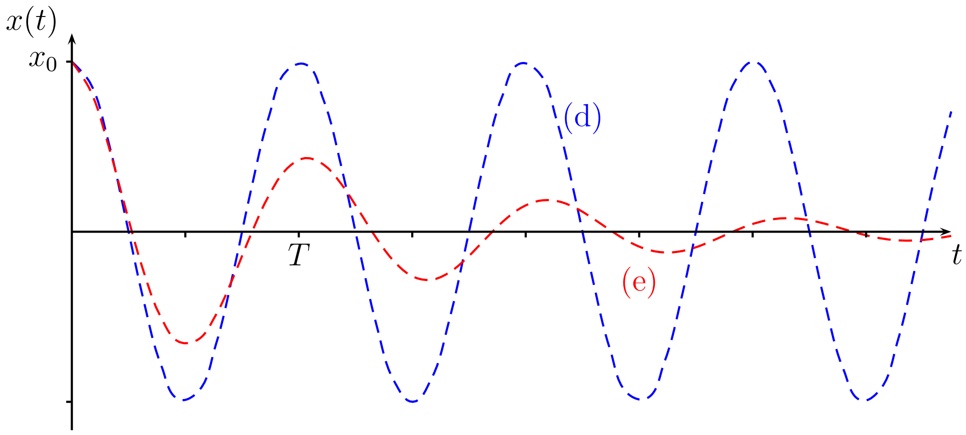
(c) The velocity at the equilibrium position can be calculated by the conservation of energy

$$\frac{1}{2}D{x_0}^2 = \frac{1}{2}mv^2 \qquad \Rightarrow \qquad \underline{\underline{v}} = x_0\sqrt{\frac{D}{m}} = 2\pi f x_0 = \underline{\underline{38\,\frac{\mathrm{cm}}{\mathrm{s}}}}$$

or by the solution

$$\begin{aligned} x(t) &= x_0\cos(\omega t) \qquad \Rightarrow \qquad v(t) = -x_0\omega\sin(\omega t) \\ \Rightarrow \qquad v &= v_{\max} = x_0\omega = 38\,\frac{\mathrm{cm}}{\mathrm{s}}\ . \end{aligned}$$

(d),(e)



2. Exercise:

A point mass m is hanging on a string with length ℓ and can move frictionless through air. At $t = 0$ the point mass is at an angle $\phi(0) = \phi_0$ to the vertical.

(a) State the equation of motion.

(b) What is the general solution $\phi(t)$ for small angles?

(c) Determine the period T of the oscillation.

(d) Find the tension force F_T in the string.

(e) With what frequency Ω should the center of rotation oscillate to get the maximum amplitude?

Solution:

(a) The equation of motion is

$$-\ell mg \sin\phi = M = \dot{L} = m\ell^2\ddot{\phi} \qquad \rightarrow \qquad \underline{\underline{\ddot{\phi} + \frac{g}{\ell}\phi = 0}} \; .$$

(b) With $\mathcal{A} = \phi_0$ and $\phi_{\text{initial}} = 0$ the solution reads

$$\underline{\underline{\phi(t) = \phi_0 \cos(\omega t)}} \qquad \text{with} \qquad \omega = \sqrt{\frac{g}{\ell}} \, .$$

(c) The period of oscillation is given by

$$\underline{\underline{T}} = \frac{2\pi}{\omega} = \underline{\underline{2\pi\sqrt{\frac{\ell}{g}}}} \; .$$

(d) The tension force is given by

$$\underline{\underline{F_T(t)}} = mg\cos(\phi(t)) = \underline{\underline{mg\cos\left(\phi_0 \cos(\omega t)\right)}} \; .$$

(e) It should oscillate with the resonant frequency given by

$$\underline{\underline{\Omega}} = \omega = \underline{\underline{\sqrt{\frac{g}{\ell}}}}\ .$$

3. Exercise:

Consider again a point mass m_p which is now put at one end of a rod with mass m_r and length ℓ while the other end is fixed. Initially the rod is at an angle $\phi(0) = \phi_0$ to the vertical. Neglect any friction.

(a) What is the total moment of inertia Θ for this kind of rotation?

(b) Determine the resulting moment of torque $|\vec{M}|$ for the system.

(c) State the equation of motion.

(d) Find the period of oscillation T in the case of small angles.

(e) What is the reduced length of the pendulum?

Solution:

(a) The moment of inertia is the sum over its components:

$$\begin{aligned}\underline{\underline{\Theta}} &= \Theta_{1p} + \Theta_r = \Theta_{1p} + \Theta_r^{\mathrm{CM}} + m_r\frac{\ell^2}{4} \\ &= m_p\ell^2 + \frac{1}{12}m_r\ell^2 + m_r\frac{\ell^2}{4} = \underline{\underline{\frac{3m_p + m_r}{3}\ell^2}}\end{aligned}$$

where for the rod the parallel axis theorem was used.

(b) The distance of the center of mass to the point of rotation is given by

$$R = \frac{m_r\ell/2 + m_p\ell}{m_p + m_r} = \frac{m_r + 2m_p}{m_r + m_p}\frac{\ell}{2}$$

When concidering gravity the rigid body can be considered as point mass at the center of mass. The moment of inertia is therefore given by

$$\underline{\underline{|\vec{M}|}} = R(m_r + m_g)g\sin\phi = \underline{\underline{\frac{m_r + 2m_p}{2}\ell g\sin\phi}}$$

Alternative, one can calculate the moment by the sum over its components:
On the point mass the moment of torque

$$|\vec{M}_p| = \ell m_p g\sin\phi$$

is acting while at the center of mass of the rod the moment of torque

$$|\vec{M}_r| = \frac{\ell}{2}m_r g\sin\phi$$

can be found. Therefore the total moment of torque is given by

$$|\vec{M}| = |\vec{M}_p| + |\vec{M}_r| = \frac{2m_p + m_r}{2}\ell g\sin\phi\,.$$

(c) Collecting the last two results the equation of motion reads

$$\Theta \dot{L} = |\vec{M}| \qquad \Rightarrow \qquad \underline{\underline{\ddot{\phi} + \frac{3g}{2\ell}\frac{2m_p + m_r}{3m_p + m_r}\sin\phi = 0}} \;.$$

(d) For small angles the equation of motion reads

$$\ddot{\phi} + \frac{3g}{2\ell}\frac{2m_p + m_r}{3m_p + m_r}\phi = 0\,.$$

Since this is the equation of motion for a harmonic oscillator the period of oscillation is given by

$$\underline{\underline{T}} = \frac{2\pi}{\omega} = \underline{\underline{2\pi\sqrt{\frac{2\ell}{3g}\frac{3m_p + m_r}{2m_p + m_r}}}} \;.$$

(e) Comparing this result with the one for the mathematical pendulum gives the reduced length of the pendulum:

$$\underline{\underline{\ell_r = \frac{2\ell}{3}\frac{3m_p + m_r}{2m_p + m_r}}}$$