KIT - International Department GmbH

Pre-Semester Physics - Exercises Summer 2010

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1. **Exercise:**
   Find the ratio of the electric and the gravitational force

   (a) between two electrons with mass $m_e = 9.1 \cdot 10^{-31}\,\text{kg}$ which are separated by a distance $r = 1\,\text{m}$.

   (b) for an isotropic ball with mass $m = 1\,\text{kg}$ charged by $q = 1\,\text{mC}$ in a thunderstorm generating an homogeneous electric field of $\mathcal{E} = 10^6\,\frac{\text{V}}{\text{m}}$ on the earth ($g = 10\,\frac{\text{m}}{\text{s}^2}$).

   Solution:

   (a) The Forces can be calculated by Coulomb's law and its gravitational analogon:

$$\begin{aligned} F_G &= G\frac{m_e^2}{r^2} = 6.67 \cdot 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2} \cdot \frac{9.1^2 \cdot 10^{-31\cdot 2}\,\text{kg}^2}{1^2\,\text{m}^2} = 5.5 \cdot 10^{-71}\,\text{N} \\ F_e &= \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{1}{4 \cdot \pi \cdot 8.85 \cdot 10^{-12}\,\frac{\text{C}}{\text{Vm}}} \cdot \frac{1.6^2 \cdot 10^{-19\cdot 2}\,\text{C}^2}{1^2\,\text{m}^2} = 2.3 \cdot 10^{-28}\,\text{N} \\ \frac{F_e}{F_G} &= \frac{2.3 \cdot 10^{-28}\,\text{N}}{5.5 \cdot 10^{-71}\,\text{N}} = 4.2 \cdot 10^{42} \end{aligned}$$

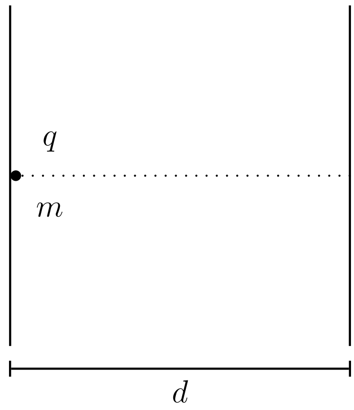
   Thus in the microscopic world the electromagnetic force is usually much stronger than the gravitational one, so that the latter one can be neglected.

   (b) Here the following simplified formulas can be used:

$$\begin{aligned} F_G &= mg = 1\,\text{kg} \cdot 10\,\frac{\text{m}}{\text{s}^2} = 10\,\text{N} \\ F_e &= q\mathcal{E} = 10^{-3}\,\text{C} \cdot 10^6\,\frac{\text{V}}{\text{m}} = 10^3\,\text{N} \\ \frac{F_e}{F_G} &= \frac{10^3\,\text{N}}{10\,\text{N}} = 10^2 \end{aligned}$$

   Therefore, for the masses and charges considered in the following the gravitational force can still be neglected. However, for charges which are smaller (and masses larger) than the one stated above both forces have to be considered.

2. Exercise:



Consider a point charge $q = 50\,\text{mC}$ with mass $m = 20\,\text{g}$ which is emitted by the left plate of a capacitor. The plates of the capacitor are separated by a distance $d = 20\,\text{m}$.

(a) How much energy gains the point charge by passing through the capacitor if a voltage of $U = 2\,\text{V}$ is applied to the capacitor?

(b) Which side has to be on the higher potential?

(c) In which direction points the electric field inside the capacitor?

(d) What voltage U_0 is needed in order to accelerate the point charge to a speed $v_0 = 20\,\frac{\text{m}}{\text{s}}$ (at the second plate)?

(e) Determine the acceleration a in the last case.

(f) How long did the point charge need in order to pass through the capacitor?

Solution:

(a) The energy is given by

$$\underline{\underline{E_e}} = qU = \underline{\underline{0.10\,\text{J}}}$$

(b) Since the positive charge should be accelerated the $\underline{\underline{\text{left plate}}}$ where it was charged has to be on the higher potential.

(c) Since the electric field points from the positive to the negative charges, i.e. from the higher to the lower potential, it points $\underline{\underline{\text{form left to right}}}$.

(d) From the conservation of energy follows

$$qU_0 = \frac{1}{2} m {v_0}^2 \qquad \Rightarrow \qquad \underline{\underline{U_0}} = \frac{m {v_0}^2}{2q} = \underline{\underline{80\,\text{V}}}$$

(e) The acceleration can be determined from the electric force

$$ma = F_e = q\mathcal{E} = q\frac{U_0}{d} \qquad \Rightarrow \qquad \underline{\underline{a}} = \frac{qU_0}{md} = \underline{\underline{10\,\frac{\text{m}}{\text{s}^2}}}$$

(f) Since it performs a linear motion with constant acceleration the time is given by

$$v = at \qquad \Rightarrow \qquad \underline{t} = \frac{v}{a} = \underline{\underline{2\,\mathrm{s}}}$$

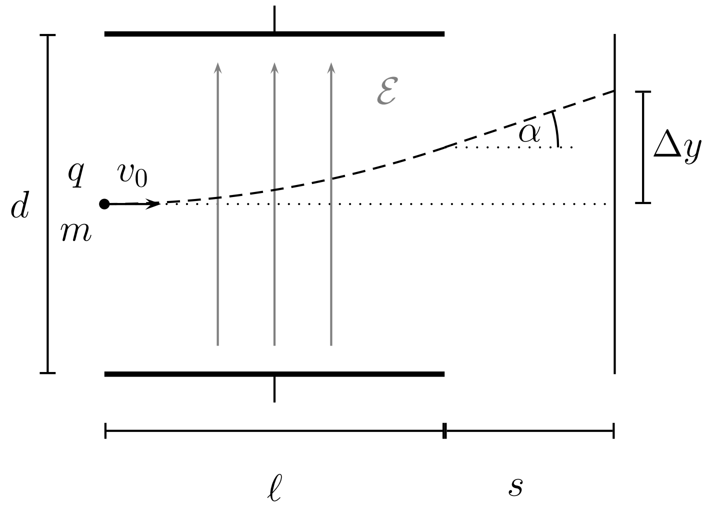
- Note that one can find the values above (f,e,d) from the equation of a linear motion with constant acceleration, as well:

$$d = \frac{1}{2}at^2, \qquad v = at \qquad \Rightarrow \qquad \frac{d}{v} = \frac{1}{2}t \qquad \Rightarrow \qquad t = \frac{2d}{v} = 2\,\mathrm{s}$$

$$v = at \qquad \Rightarrow \qquad a = \frac{v}{t} = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$ma = q\mathcal{E} = q\frac{U_0}{d} \qquad \Rightarrow \qquad U_0 = \frac{mad}{q} = 80\,\mathrm{V}$$

3. Exercise:



Consider now a point charge $q = 10\,\mathrm{mC}$ with mass $m = 5\,\mathrm{g}$ which moves from the left inside a parallel plate capacitor with velocity $v_0 = 6\,\frac{\mathrm{m}}{\mathrm{s}}$ parallel to the plates and just in the center of the capacitor. The parallel plates of the capacitor are squares which are separated by a distance $d = 7\,\mathrm{m}$ and have a length $\ell = 3\,\mathrm{m}$. There is an electric field $\mathcal{E} = 20\,\frac{\mathrm{V}}{\mathrm{m}}$ present which points upwards in the region between the plates. A screen is placed perpendicular to the plates at a distance $s = 7\,\mathrm{m}$ from the end of the plates.

(a) At which voltage U lies the capacitor?

(b) In which direction is the point charge deflected, i.e. is it correctly drawn in the figure?

(c) What is the acceleration a of the point charge?

(d) How long does the point charge need in order to pass through the parallel plate capacitor?

(e) What is its velocity v then?

(f) Determine the angle α with which the point charge leaves the capacitor.

(g) Find the displacement Δy of the point charge at the screen measured from the axis of the motion before the capacitor.

Solution:

(a) The field inside the capacitor is determined by the applied voltage:

$$\mathcal{E} = \frac{U}{d} \qquad \Rightarrow \qquad \underline{\underline{U}} = \mathcal{E} d = \underline{\underline{140\,\mathrm{V}}}$$

(b) The positive point charge will travel along the lines of the electric field, i.e. $\underline{\underline{\text{ upwards }}}$.

(c) From the electric force the acceleration can be determined to

$$ma = F_e = q\mathcal{E} \qquad \Rightarrow \qquad \underline{\underline{a}} = \frac{q}{m}\mathcal{E} = \underline{\underline{40\,\frac{\mathrm{m}}{\mathrm{s}^2}}}$$

(d) Since the acceleration only acts along the direction y perpendicular to the plates while the initial velocity is pointing parallel to them along the direction x, the position is given by

$$x(t) = v_0 t$$
$$y(t) = \frac{1}{2} a t^2$$

measured from the initial position. Therefore the time t_c is given by the first equation and the condition $x(t_c) = \ell$:

$$\underline{\underline{t_c}} = \frac{\ell}{v_0} = \underline{\underline{0.5\,\mathrm{s}}}$$

(e) The velocity is determined by the component along the x and y direction:

$$\underline{\underline{v(t_c)}} = \sqrt{v_x(t_c)^2 + v_y(t_c)^2} = \sqrt{v_0{}^2 + a^2 t_c{}^2} = \underline{\underline{20.9\,\frac{\mathrm{m}}{\mathrm{s}}}}$$

(f) The angle is determined by the components of the velocity:

$$\tan(\alpha) = \frac{v_y}{v_x} = \frac{a t_c}{v_0} \qquad \Rightarrow \qquad \underline{\underline{\alpha = 73^\circ}}$$

(g) The displacement is the sum of the displacement in the capacitor (due to the accelerated motion) and the uniform motion afterwards:

$$\underline{\underline{\Delta y = y(t_c) + s\tan(\alpha) = \frac{1}{2}a{t_c}^2 + \frac{a t_c s}{v_0} = \frac{v_0 t_c + 2s}{2v_0}a t_c = 28\,\mathrm{m}}}$$

- Of course, if the deviation in y-direction is calculated one finds a value larger than the distance of the point charge to the plates. Therefore the time when the point charge leaves the capacitor is determined by the second equation in (d):

$$\frac{d}{2} = \frac{1}{2}a{t_i}^2 \qquad \Rightarrow \qquad t_i = \sqrt{\frac{d}{a}} = 0.42\,\mathrm{s} < t_c$$

This means an impact velocity and angle of

$$v(t_i) = \sqrt{{v_0}^2 + a^2{t_i}^2} = 18\,\frac{\mathrm{m}}{\mathrm{s}} \qquad \Rightarrow \qquad \alpha = 70^\circ\,.$$

Now one can either assume that the charge can pass the plates leading to a displacement at the screen of

$$\Delta y = y(t_i) + (s + \ell - x(t_i))\tan(\alpha) = 24\,\mathrm{m}$$

or that it adds to the charges on the capacitor, leading to a change in the measured voltage to

$$U' = \mathcal{E}'d = \frac{Q'}{\epsilon_0 A}d = \frac{|Q| - q}{\epsilon_0 A}d = \frac{\epsilon_0\mathcal{E}A - q}{\epsilon_0 A} = \mathcal{E}d - \frac{qd}{\epsilon_0 A} = U - \frac{qd}{\epsilon_0\ell^2} = -8.8\cdot 10^8\,\mathrm{V}\,!$$

The negative sign therein means that afterwards the lower plate will be on the lower potential.