KIT - International Department GmbH

Pre-Semester Physics - Exercises **Summer 2010**

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1. **Exercise:**
A Nichrome wire ($\rho_s = 1.0 \cdot 10^{-6}\,\Omega\text{m}$) has a radius of $r = 6.5\,\text{mm}$.

(a) What length ℓ of a wire is needed to obtain a resistance of $R = 20\,\text{m}\Omega$?

(b) Find the magnitude of the electric field $\mathcal{E}$ in the wire if it is carrying a current of $I = 1.3\,\text{A}$.

(c) What would be the capacitance C of a similar isolating wire made of Barium titanate with $\epsilon_r = 4000$ (and metallic bases)?

(d) How many charges Q would be on its bases if the same electric field is present?

Solution:

(a) From the definition of the specific resistance it is found:

$$R = \rho_s \frac{\ell}{\pi r^2} \qquad \Rightarrow \qquad \underline{\underline{\ell}} = \frac{R\pi r^2}{\rho_s} = \underline{\underline{2.7\,\text{m}}}$$

(b) The electric field is given by

$$\underline{\underline{\mathcal{E}}} = \frac{U}{\ell} = \frac{IR}{\ell} = \frac{I\rho_s}{\pi r^2} = \underline{\underline{9.8\,\frac{\text{mV}}{\text{m}}}} \ .$$

(c) The capacitance is determined to

$$\underline{\underline{C}} = \epsilon_0 \epsilon_r \frac{\pi r^2}{\ell} = 1.8 \cdot 10^{-12}\,\text{F} = \underline{\underline{1.8\,\text{pF}}} \ .$$

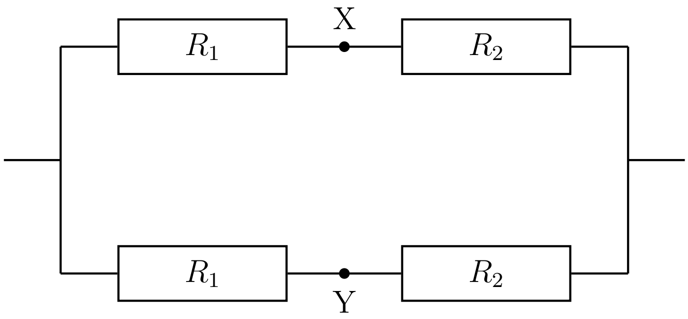
(d) By the definition of the capacitance a charge of

$$C = \frac{Q}{U}, \qquad U = \mathcal{E}\ell \qquad \Rightarrow \qquad \underline{\underline{Q}} = \mathcal{E}\ell C = \underline{\underline{4.6 \cdot 10^{-14}\,\text{C}}}$$

is needed.

2. Exercise:

Consider the following resistor network:



The resistors have the resistances $R_1 = 3.0\,\Omega$ and $R_2 = 2.0\,\Omega$.

(a) Determine the equivalent resistance R_{tot}.

(b) What would be the current I_0 through the whole circuit if a voltage of $U_0 = 1.5\,\text{V}$ is applied?

(c) How large is the current I_1 which flows through a resistor with resistance R_1 in this case?

(d) Find the voltages U_1 and U_2 which drop at the resistors with resistance R_1 and R_2.

(e) What powers P_1 and P_2 are dissipated at the resistors R_1 and R_2?

(f) Determine the electrical potential at the points X and Y if the left end of the circuit lies at the potential of $\phi_0 = 2.0\,\text{V}$ while the right end is at a lower value.

(g) Which voltage U_{XY} drops between the points X and Y and how large is the current I_{XY} which flows in a wire connecting these points?

Solution:

(a) The total / equivalent resistance is given by

$$\underline{\underline{R_{\text{tot}}}} = \frac{1}{\frac{1}{R_1+R_2} + \frac{1}{R_1+R_2}} = \frac{R_1+R_2}{2} \underline{\underline{2.5\,\Omega}} \; .$$

(b) The current can be determined by Ohm's law:

$$R_{\text{tot}} = \frac{U_0}{I_0} \qquad \Rightarrow \qquad \underline{\underline{I_0}} = \frac{U_0}{R_{\text{tot}}} = \underline{\underline{0.60\,\text{A}}}$$

(c) Since the voltage does not change if three wires connect each other and the current which flows through R_1 is the same as the one which flows through R_2 the current through R_1 is found to be

$$R_1 + R_2 = \frac{U_0}{I_1} \qquad \Rightarrow \qquad \underline{\underline{I_1}} = \frac{U_0}{R_1 + R_2} = \underline{\underline{0.30\,\text{A}}} \; .$$

(d) The voltages can be calculated by Ohm's law. They can be checked by Kirchhoff's voltage law:

$$
\begin{aligned}
R_1 &= \frac{U_1}{I_1} \qquad \Rightarrow \qquad \underline{\underline{U_1}} = R_1 I_1 = \underline{\underline{0.90\,\mathrm{V}}} \\
R_2 &= \frac{U_2}{I_1} \qquad \Rightarrow \qquad \underline{\underline{U_2}} = R_2 I_1 = \underline{\underline{0.60\,\mathrm{V}}} \\
U_1 + U_2 &\overset{!}{=} U_0 \qquad \checkmark
\end{aligned}
$$

(e) By the definition of the electric power the values can be found. They can be checked by the conservation of energy:

$$
\begin{aligned}
\underline{\underline{P_1}} &= U_1 I_1 = \underline{\underline{0.27\,\mathrm{W}}} \\
\underline{\underline{P_2}} &= U_2 I_1 = \underline{\underline{0.18\,\mathrm{W}}} \\
2(P_1 + P_2) &\overset{!}{=} U_0 I_0 = 0.90\,\mathrm{W} \qquad \checkmark
\end{aligned}
$$

(f) Since the voltage is the difference in the electrical potential which drops at R_1 the result is given by

$$\underline{\underline{\phi_\mathrm{X} = \phi_\mathrm{Y}}} = \phi_0 - U_1 = \underline{\underline{1.1\,\mathrm{V}}}\ .$$

It can be checked if the potential in the rest of the circuit is determined:

$$\phi_\mathrm{X} - U_2 = 0.5\,\mathrm{V} \overset{!}{=} \phi_0 - U_0 \qquad \checkmark$$

(g) Since both points are lying on the same electrical potential there can not be a voltage drop. However, if there is no voltage drop no current will flow:

$$\underline{\underline{U_\mathrm{XY} = 0}} \qquad \underline{\underline{I_\mathrm{XY} = 0}}$$

Note:

The Vacuum permittivity is given by $\epsilon_0 = 8.85 \cdot 10^{-12}\,\frac{\mathrm{As}}{\mathrm{Vm}}$