KIT - International Department GmbH

Pre-Semester Physics - Exercises **Summer 2010**

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1. Exercise:

A point charge $q = 50\,\text{mC}$ with mass $m = 20\,\text{g}$ is moving with velocity $v = 8.0\,\frac{\text{m}}{\text{s}}$ (perpendicularly) towards the edge of a large quadratic region where a homogeneous magnetic field $B = 0.25\,\text{T}$ is present. Consider a velocity perpendicular to the magnetic field at first.

(a) At what distance d to the entry point will the point charge leave the region?

(b) How long does it remain in the region?

(c) State the electric field $\mathcal{E}$ perpendicular to the magnetic one (and in the region) so that the point charge is not deviated from its course.

(d) What kind of motion does the point charge perform if its velocity is at an angle $\alpha = 60°$ to the magnetic field (with no electric field present)?

(e) Calculate the time $\tilde{t}$ it spends inside the region for the non-perpendicular velocity of (d).

(f) Find again the distance $\tilde{d}$ to the entry point when it leaves the region for this case.

Solution:

(a) Since the point charge will perform a circular motion where the radius is given by the balance of the magnetic and centripetal force

$$qvB = F_B = F_C = \frac{mv^2}{r} \qquad \Rightarrow \qquad r = \frac{mv}{qB}$$

the distance is given by twice this value

$$\underline{\underline{d}} = 2r = \frac{2mv}{qB} = \underline{\underline{26\,\text{m}}}$$

(The point charge is assumed to be far away from the corners of the region.)

(b) The period is therefore given by

$$\underline{\underline{t}} = \frac{T}{2} = \frac{\pi}{\omega} = \frac{\pi r}{v} = \frac{\pi m}{qB} = \underline{\underline{5.0\,\text{s}}}\,.$$

(c) From the balance of the electrical and magnetic force the following result is obtained:

$$q\mathcal{E} = qvB \qquad \Rightarrow \qquad \underline{\underline{\mathcal{E}}} = vB = \underline{\underline{2\,\frac{\text{N}}{\text{C}}}}$$

(d) The velocity can be decomposed in a component perpendicular $v_\perp = v\sin(\alpha)$ and a velocity parallel $v_\parallel = v\cos(\alpha)$ to the magnetic field. While the motion along $v_\parallel$ is not affected by the magnetic field the component perpendicular would lead to a circular motion in the plane moving with $v_\parallel$. Therefore it would perform circles which are shifted continuously in the direction of the magnetic field (if the point charge would not leaving the region). Since this is like a helix it is called a helical motion.

(e) Since the radius of this helix can be determined when looking at its projection along its axis only the velocity $v_\perp$ enters. However, since in the formula above the velocity does not enter the time until it leaves the region does not change:

$$\underline{\tilde{t}} = \frac{\tilde{T}}{2} = \frac{\pi}{\tilde{\omega}} = \frac{\pi r}{v\sin(\alpha)} = \frac{\pi m}{qB} = \underline{\underline{5.0\,\mathrm{s}}}$$

(f) While the radius has to be modified due to the different effective velocity $v_\perp$ the displacement caused by the uniform motion with $v_\parallel$ has to be added:

$$\underline{\tilde{d}} = \sqrt{4\frac{m^2v^2\sin^2(\alpha)}{q^2B^2} + v^2\cos^2(\alpha)\tilde{t}^2} = 2\frac{mv}{qB}\sqrt{\sin^2(\alpha) + \frac{\pi^2}{4}\cos^2(\alpha)} = \underline{\underline{30\,\mathrm{m}}}$$

2. Exercise:

Consider a coil with $N = 100$ turns, length $\ell = 12.6\,\mathrm{cm}$ and radius $r = 1.0\,\mathrm{cm}$ through which a current of $I = 1.0\,\mathrm{A}$ flows. A frame is moving towards the coil with velocity $v = 1.0\,\frac{\mathrm{m}}{\mathrm{s}}$ parallel to two of its edges. At $t = 0\,\mathrm{s}$ its first edge passes the turns. The metallic, quadratic frame with edge length $a = 1.0\,\mathrm{mm}$ is oriented perpendicular to the axis of the coil. (Neglect the curvature of the turns, the effects of self induction as well as the magnetic field outside the coil.)

(a) Calculate the magnetic field of the coil.

(b) How would this quantity change when iron ($\mu_r = 1000$) is inserted into the coil?

(c) Sketch the magnetic flux $\Phi(t)$ through the frame in the case of the empty coil.

(d) What is the maximal induced voltage $\hat{U}_{\mathrm{ind}}$ in the frame in this case?

(e) When can this value be measured?

Solution:

(a) Using the formula derived in the lecture, the magnetic field is given by

$$\underline{\underline{B}} = \mu_0 \frac{N}{\ell} I = \underline{\underline{1\,\text{mT}}}$$

(b) The magnetic field enhances to

$$\underline{\underline{B_r}} = \mu_r B = \underline{\underline{1\,\text{T}}}$$

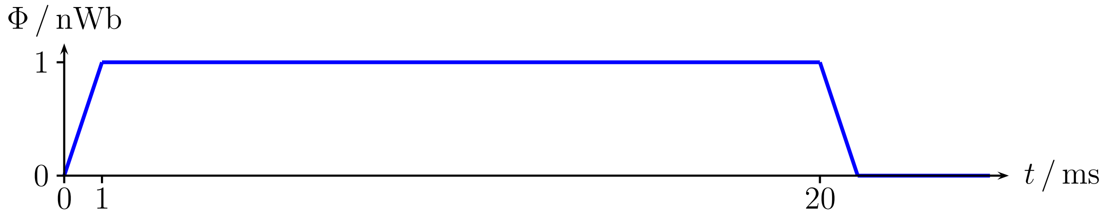
(c) The magnetic flux for a homogeneous field and an increasing area is given by

$$\Phi(t) = BA = Bavt = 1\,\mu\text{V} \cdot t \quad \text{for } 0 < t < \frac{a}{v} = 1\,\text{ms}.$$

After that last time index the flux will be constant $\Phi(t) = 1\,\text{nWb}$ until it will start to decrease due to a decreasing area:

$$\Phi(t) = 1\,\text{ns} - 1\,\mu\text{V}(t - 20\,\text{ms}) \quad \text{for } \frac{2r}{v} = 20\,\text{ms} < t < \frac{2r}{v} + \frac{a}{v} = 21\,\text{ms}$$

Otherwise it will vanish.



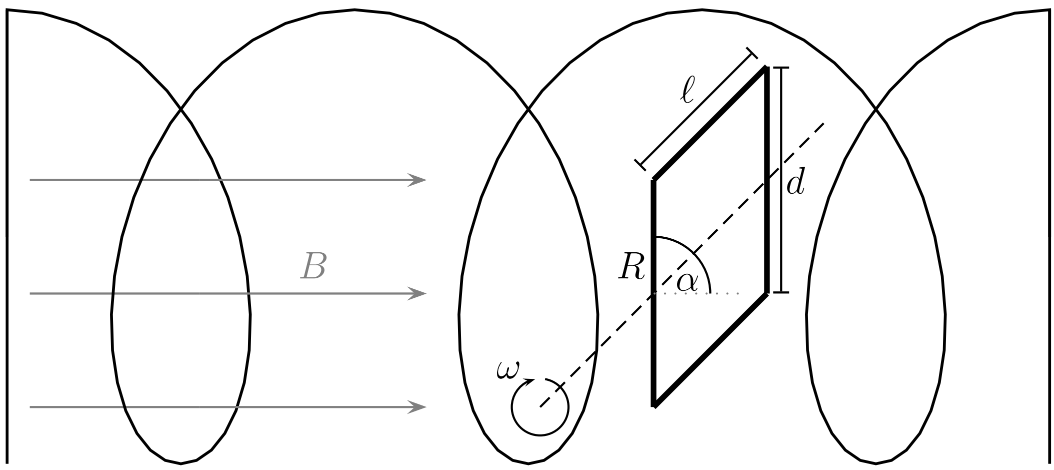
(d) Since the orientated voltage is

$$U_{\text{ind}} = -\dot{\Phi} = \begin{cases} -1\,\mu\text{V} & \text{for} \quad 0\,\text{ms} < t < 1\,\text{ms} \\ 1\,\mu\text{V} & \text{for} \quad 20\,\text{ms} < t < 21\,\text{ms} \\ 0\,\mu\text{V} & \text{else} \end{cases}$$

the maximal voltage is $\underline{\underline{\hat{U}_{\text{ind}} = 1\,\mu\text{V}}}$.

(e) It can be measured for from $\underline{\underline{0\,\text{ms to } 1\,\text{ms}}}$ and from $\underline{\underline{20\,\text{ms to } 21\,\text{ms}}}$.

3. Exercise:



Inside a coil, which produces a magnetic field B a metal frame is rotated at constant angular velocity ω. The edges of the rectangular frame consist of a wire with length ℓ and d where the edge with length ℓ is always perpendicular to the magnetic field inside the coil. The frame has a total resistance of R. Initially, the frame is parallel to the turns of the coil.

(a) State the time-dependence of the angle α between the frame and the magnetic field.

(b) Find the magnetic flux Φ through the frame as function of time t.

(c) Determine the induced voltage $U_{\text{ind}}(t)$ inside the frame.

(d) What is the current $I(t)$ in the frame?

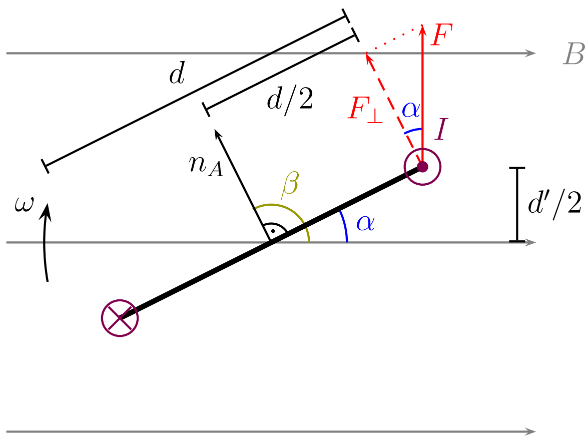
(e) Calculate the moment of torque $M(t)$ necessary to keep the frame at constant angular velocity.

(f) What power $P(t)$ is put into the system by this moment of torque?

Solution:

(a) Since it is rotated with constant angular velocity ω the angle α is given by

$$\dot{\alpha} = -\omega \qquad \Rightarrow \qquad \underline{\underline{\alpha = \frac{\pi}{2} - \omega t}} \ .$$



(b) By the sinus of this angle the projection d' of the length d perpendicular to the magnetic field is reduced, leading to a magnetic flux

$$\underline{\underline{\Phi(t)}} = \vec{B}\cdot\vec{A} = BA\cos(\beta) = B\ell\underbrace{d\cos\left(\frac{\pi}{2}+\alpha\right)}_{d'} = B\ell d\cos(\pi-\omega t) = \underline{\underline{-B\ell d\cos(\omega t)}} \ .$$

(c) The induced voltage is determined by its time derivative

$$\underline{\underline{U_{\text{ind}}}}(t) = -\dot{\Phi}(t) = \underline{\underline{-B\ell d\omega\sin(\omega t)}} \ .$$

(d) From Ohm's law the current can be obtained:

$$R = \frac{U_{\text{ind}(t)}}{I(t)} \qquad \Rightarrow \qquad \underline{\underline{I(t)}} = \frac{U_{\text{ind}(t)}}{R} = \underline{\underline{-\frac{B\ell d}{R}\omega\sin(\omega t)}}$$

(e) The force on one edge is given by

$$F(t) = B\,I(t)\,\ell = -\frac{B^2\ell^2 d}{R}\omega\sin(\omega t)\,.$$

However, this force is only perpendicular to the edge with length d at the initial position (and on positions after $T/2,\ T,\ldots$). In any other times it is at an angle $\alpha = \frac{\pi}{2}-\omega t$. Therefore, the induced moment of torque contributed from the two edges with length ℓ is determined by

$$\begin{aligned}\underline{\underline{M(t)}} &= 2\cdot\frac{d}{2}\cdot F_\perp = dF\cdot\cos(\alpha)\\ &= d\frac{-B^2\ell^2 d}{R}\omega\sin(\omega t)\cdot(-1)\sin(\omega t) = \underline{\underline{\frac{B^2\ell^2 d^2}{R}\omega\sin^2(\omega t)}} \ .\end{aligned}$$

This induced moment of torque has to be compensated by the applied one in order to get a resulting vanishing moment of torque necessary for a constant angular velocity.

(f) The power provided by the moment of torque is given by

$$\underline{\underline{P(t)}} = M(t)\omega = \underline{\underline{\frac{B^2\ell^2 d^2}{R}\omega^2\sin^2(\omega t)}} \ .$$

It has the same value as the power dissipated in the resistance of the frame:

$$P_{\text{el}}(t) = U_{\text{ind}}(t)I(t) = \frac{B^2\ell^2 d^2}{R}\omega^2\sin^2(\omega t) \qquad \checkmark$$

Note:

The vacuum permeability is given by $\mu_0 = 1.26\cdot 10^{-6}\,\frac{\text{J}}{\text{A}^2\text{m}}$