KIT - International Department GmbH

Pre-Semester Physics - Exercises **Summer 2010**

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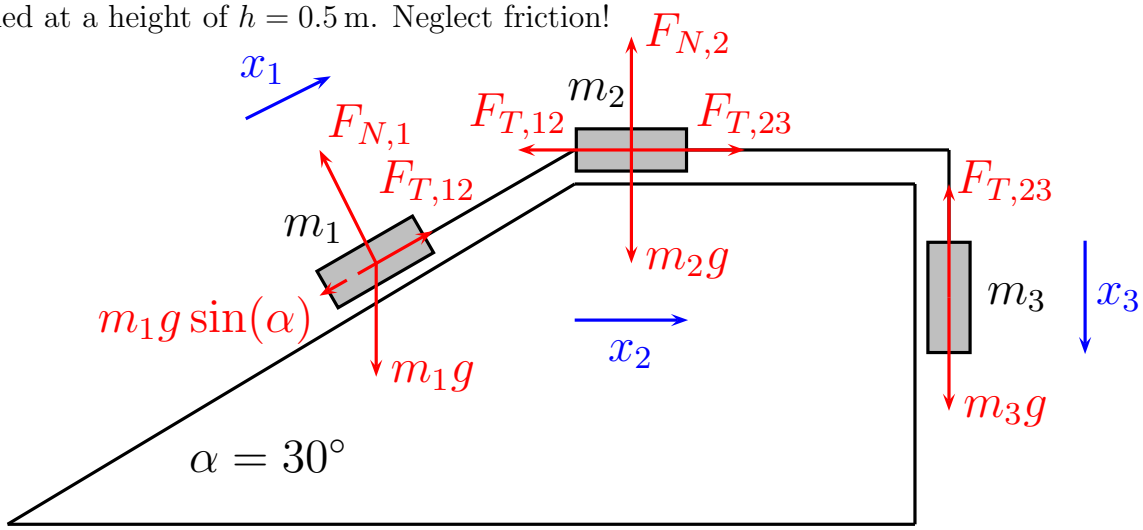
Extra Sheet II
25.8.2010

1. Exercise:

Three masses $m_1 = m_3 = 1\,\text{kg}$ and $m_2 = 10\,m_1$ are arranged as shown in the figure below. They are all exposed to the gravitational force ($g = 10\,\frac{\text{m}}{\text{s}^2}$). The mass m_3 is positioned at a height of $h = 0.5\,\text{m}$. Neglect friction!



(a) Draw all elementary forces acting on the masses in the picture with straight arrows. Be careful to distinguish different tension forces.

(b) Decompose the forces in the picture into forces where the bodies can move by drawing dashed arrows.

(c) Specify a coordinate system x_1, x_2, x_3 for each mass by drawing an arrow along the moving direction.

(d) State the forces in which direction the body can move as formulas.

$$\boxed{F_{D,1} = F_{T,12} - m_1 g \sin(\alpha)}$$

$$\boxed{F_{D,2} = -F_{T,12} + F_{T,23}}$$

$$\boxed{F_{D,3} = m_3 g - F_{T,23}}$$

(e) Write down the equations of motion for each mass.

$$\boxed{m_1 a_1 = F_{T,12} - m_1 g \sin(\alpha)}$$

$$\boxed{m_2 a_2 = -F_{T,12} + F_{T,23}}$$

$$\boxed{m_3 a_3 = m_3 g - F_{T,23}}$$

(f) How are the positions connected?

$$\boxed{x_1 = +x_2 = +x_3}$$

(g) Replace in the equations of motion the acceleration of the different masses by the common acceleration of the system.

$$\boxed{m_1 a = F_{T,12} - m_1 g \sin(\alpha)}$$

$$\boxed{m_2 a = -F_{T,12} + F_{T,23}}$$

$$\boxed{m_3 a = m_3 g - F_{T,23}}$$

(h) Solve two of the equation of motions to get the tension forces.

$$\boxed{F_{T,12} = m_1 a + m_1 g \sin(\alpha)}$$

$$\boxed{F_{T,23} = m_3 g - m_3 a}$$

(i) Put them into the last equation and determine the acceleration a of the system.

$$\boxed{m_2 a = -m_1 a - m_1 g \sin(\alpha) + m_3 g - m_3 a \qquad \Rightarrow \qquad a = \frac{m_3 - m_1 \sin(\alpha)}{m_1 + m_2 + m_3} g = 0.42 \, \frac{\mathrm{m}}{\mathrm{s}^2}}$$

(j) What motion performs the mass m_3?

☐ uniform motion ☒ motion with constant acceleration ☐ circular motion

(k) How is the distance traveled of the mass m_3 connected to the acceleration of the system? Specify all constants of the motion.

$$\boxed{x_3(t) = \tfrac{1}{2}at^2 = 0.21\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot t^2}$$

(l) Find the time t, the mass m_3 needs to hit the ground.

$$\boxed{h = \tfrac{1}{2}at^2 \qquad \Rightarrow \qquad t = \sqrt{\tfrac{2h}{a}} = 1.5\,\mathrm{s}}$$

(m) Put the determined acceleration into the two equation of motions you solved for the tension forces in order to answer the question "What is the tension force in the two ropes?"

$$\boxed{F_{T,12} = 5.4\,\mathrm{N}}$$

$$\boxed{F_{T,23} = 9.6\,\mathrm{N}}$$

(n) What ratio of m_1 and m_3 is needed, such that m_2 stays at rest?

$$\boxed{a = 0 \qquad \Rightarrow \qquad m_3 - m_1 \sin(\alpha) = 0 \qquad \Rightarrow \qquad \tfrac{m_1}{m_3} = \tfrac{1}{\sin(\alpha)} = 2}$$