Pre-Semester Physics - Exercises Summer 2010

Stefan Kremer
stefan.kremer@ensicaen.fr

Extra Sheet III
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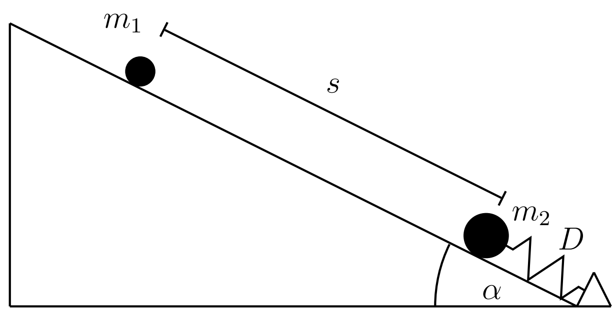
1. Exercise:



Consider two masses $m_1 = 1.0\,\mathrm{kg}$ and $m_2 > m_1$ at distance $s = 12\,\mathrm{m}$ on a frictionless inclined plane with angle $\alpha = 45°$. Both masses are initially at rest but are subject to gravity ($g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$). The position of the second mass m_2 is fixed by a spring with constant D. The goal is to find the maximum height h_1 of the mass m_1 and the maximum compression s_2 of the spring after a total elastic collision.

(a) What would be the downward force acting on the mass m_1?

$$\boxed{F_D = m_1 g \sin(\alpha) = 7.1\,\mathrm{N}}$$

(b) From Newton's 2nd axiom state the acceleration of the mass m_1

$$\boxed{a = \frac{F_D}{m_1} = 7.1\,\frac{\mathrm{m}}{\mathrm{s}^2}}$$

(c) This means that the first mass performs a

☐ uniform motion ☒ motion with constant acceleration ☐ circular motion

(d) How depends the position of the first mass in general?

$$\boxed{x(t) = \frac{1}{2}at^2 + v_0 t + x_0}$$

(e) Specify the integration variables.

$$\boxed{v_0 = 0}$$

$$\boxed{x_0 = 0 \qquad \text{(dep. on coord.sys.)}}$$

(f) What would be the position of the mass m_1 at the collision for this choice?

$$\boxed{x(t) = s = 12\,\mathrm{m} \qquad \text{(dep. on coord.sys.)}}$$

(g) Solve the equation for the position for this value to the time in order to determine the time t when both masses collide.

$$\boxed{t = \sqrt{\tfrac{2s}{a}} = 1.8\,\mathrm{s}}$$

(h) Find the velocity v before the collision by taking the time derivative of the position function and put the last value in.

$$\boxed{v = at = 13\,\tfrac{\mathrm{m}}{\mathrm{s}}}$$

(i) State the laws of momentum and energy conservation for the elastic collision in the case that the second mass is at initially at rest.

$$\boxed{m_1 v = m_1 v_1' + m_2 v_2'}$$

$$\boxed{\tfrac{1}{2} m_1 v^2 = \tfrac{1}{2} m_1 {v_1'}^2 + \tfrac{1}{2} m_2 {v_2'}^2}$$

(j) Solve the law of momentum conservation to v_2', the velocity of the second mass m_2 after the collision.

$$\boxed{v_2' = \tfrac{m_1}{m_2}(v - v_1')}$$

(k) Put this expression into the law of energy conversion and simplify it to a quadratic expression for v_1', the velocity of the mass m_1 after the collision.

$$\boxed{0 = \left(m_1 + \tfrac{{m_1}^2}{m_2}\right) \cdot {v_1'}^2 - 2\tfrac{{m_1}^2}{m_2} v \cdot v_1' + \left(\tfrac{{m_1}^2}{m_2} - m_1\right) v^2}$$

(l) Use the solution formula of quadratic expressions to obtain two solutions. Simplify them!

$$(1):\quad \boxed{v'_{1,1} = v}$$

$$(2):\quad \boxed{v'_{1,2} = \frac{m_1 - m_2}{m_1 + m_2} v}$$

(m) Which solution describes the situation before the collision?

$$\boxed{(1) \qquad \text{(dep. on choice)}}$$

(n) Take the other expression and evaluate it for $m_2 = 2\,\mathrm{kg}$ and the given values.

$$\boxed{v'_1 = -4.3\,\frac{\mathrm{m}}{\mathrm{s}}}$$

(o) Put this value in the formula for v'_2 in order to determine the velocity of the second mass after the collision.

$$\boxed{v'_2 = 8.7\,\frac{\mathrm{m}}{\mathrm{s}}}$$

(p) Now state the law of energy conversion which determines the maximum height h_1 of the first mass if it has a velocity v'_1. Take as zero point of the potential energy the one at the position of the collision.

$$\boxed{\frac{1}{2} m_1 {v'_1}^2 = m_1 g h_1}$$

(q) Solve this equation to h_1.

$$\boxed{h_1 = \frac{{v'_1}^2}{2g}}$$

(r) Put in the values calculated for v'_1.

$$\boxed{\underline{\underline{h_1 = 0.94\,\mathrm{m}}}}$$

(s) Write down the law of energy conversion for the second mass when hitting the spring on the inclined plane if it has the velocity v'_2. Do not forget the potential energy caused by the difference in height h_2.

$$\boxed{\frac{1}{2} m_2 {v'_2}^2 = \frac{1}{2} D {s_2}^2 - m_2 g h_2}$$

(t) Express therein the difference in height by the compression of the spring.

$$\boxed{\frac{1}{2} m_2 {v'_2}^2 = \frac{1}{2} D {s_2}^2 - m_2 g s_2 \sin(\alpha)}$$

(u) Rewrite this equation to a quadratic expression for the compression s_2.

$$\boxed{0 = {s_2}^2 - 2\tfrac{m_2}{D} g \sin(\alpha) \cdot s_2 - \tfrac{m_2}{D} {v_2'}^2}$$

(v) Solve the equation to the two mathematical solutions.

$$\text{(I)}: \quad \boxed{s_{2,1} = \tfrac{m_2}{g} g \sin(\alpha) + \sqrt{\tfrac{{m_2}^2}{D^2} g^2 \sin^2(\alpha) + \tfrac{m_2}{D} {v_2'}^2}}$$

$$\text{(II)}: \quad \boxed{s_{2,1} = \tfrac{m_2}{g} g \sin(\alpha) - \sqrt{\tfrac{{m_2}^2}{D^2} g^2 \sin^2(\alpha) + \tfrac{m_2}{D} {v_2'}^2}}$$

(w) Which solution is the physical solution?

(I) (dep. on choice)

(x) Put in this expression the spring constant $D = 10^2\,\frac{\mathrm{N}}{\mathrm{m}}$ and the values determined above.

$$\boxed{\underline{\underline{s_2 = 1.2\,\mathrm{m}}}}$$