KIT - International Department GmbH

Pre-Semester Physics - Exercises   Summer 2010

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Extra Sheet IV
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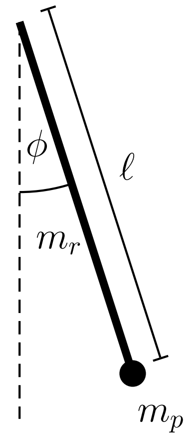
1. Exercise:



A point mass m_p is put at one end of a rod with mass $m_r = 0.6\,m_p$ and length ℓ while the other end is fixed. Initially the rod is at rest and put at an angle $\phi(0) = \phi_0$ to the vertical. The goal is to determine the function of the angle ϕ with respect to the time t.

(a) At first determine the total moment of inertia. For this reason state the moment of inertia for the rod Θ_r and the point mass Θ_p with respect to the axis of rotation separately. Make use of the fact that the moment of inertia for a long, uniform rod of mass m_{rod} and length ℓ_{rod} with respect to an axis of rotation through the center of mass is given by $\Theta_{\text{CM}} = \frac{1}{12} m_{\text{rod}} {\ell_{\text{rod}}}^2$.

$$\boxed{\Theta_r = \frac{1}{3} m_r \ell^2}$$

$$\boxed{\Theta_p = m_p \ell^2}$$

(b) Add them up in order to find the total moment of inertia dependent on the mass m_p and the length ℓ.

$$\boxed{\Theta_{\text{tot}} = \tfrac{6}{5} m_p \ell^2}$$

(c) Preliminary to the determination of the moment of torque state its value at the vertical position.

$$\boxed{M(\phi = 0) = 0}$$

(d) Find the position of the center of mass for the whole rigid body by stating its distance from the rotation axis. Simplify it so that it is only dependent on the mass m_p and the length ℓ!

$$\boxed{R = \tfrac{13}{16} \ell}$$

(e) What is the total mass (dependent on m_p)?

$$\boxed{\mathcal{M} = \tfrac{8}{5} m_p}$$

(f) State the moment of torque M as function of the angle ϕ, the mass m_p, the length ℓ and earth's gravity g.

$$\boxed{M(\phi) = \tfrac{13}{10} \ell m_p g \sin(\phi)}$$

(g) Put in $\phi = 0$ to see if it fulfils the condition mention above.

$$\boxed{M(\phi = 0) = \tfrac{13}{10} \ell m_p g \cdot 0 = 0}$$

(h) Write down the equation of motion and make sure that it describes the expected oscillation. Simplify it!

$$\boxed{\ddot{\phi} = -\tfrac{13}{12} \tfrac{g}{\ell} \sin(\phi)}$$

(i) Simplify it further in the case of small angles.

$$\boxed{\ddot{\phi} = -\tfrac{13}{12} \tfrac{g}{\ell} \phi}$$

(j) Identify this equation as the one of an harmonic oscillator and find the (angular) frequency.

$$\boxed{\omega = \sqrt{\tfrac{13}{12} \tfrac{g}{\ell}}}$$

(k) Try to remember the general solution of this differential equation.

$$\boxed{\phi(t) = \mathcal{A}\cos(\omega t + \tilde{\phi})}$$

(l) Check it by calculating the second time derivative and putting it as well as the solution into the equation of motion.

$$\boxed{\dot{\phi}(t) = -\mathcal{A}\omega\sin(\omega t + \tilde{\phi})}$$

$$\boxed{\ddot{\phi}(t) = -\mathcal{A}\omega^2\cos(\omega t + \tilde{\phi})}$$

$$\boxed{-\mathcal{A}\omega^2\cos(\omega t + \tilde{\phi}) = -\tfrac{13}{12}\tfrac{g}{\ell}\mathcal{A}\cos(\omega t + \tilde{\phi}) \qquad \Rightarrow \qquad \omega^2 = \tfrac{13}{12}\tfrac{g}{\ell}}$$

(m) Determine the general integration parameters for the given situation.

$$\boxed{\mathcal{A} = \phi_0}$$

$$\boxed{\tilde{\phi} = 0}$$

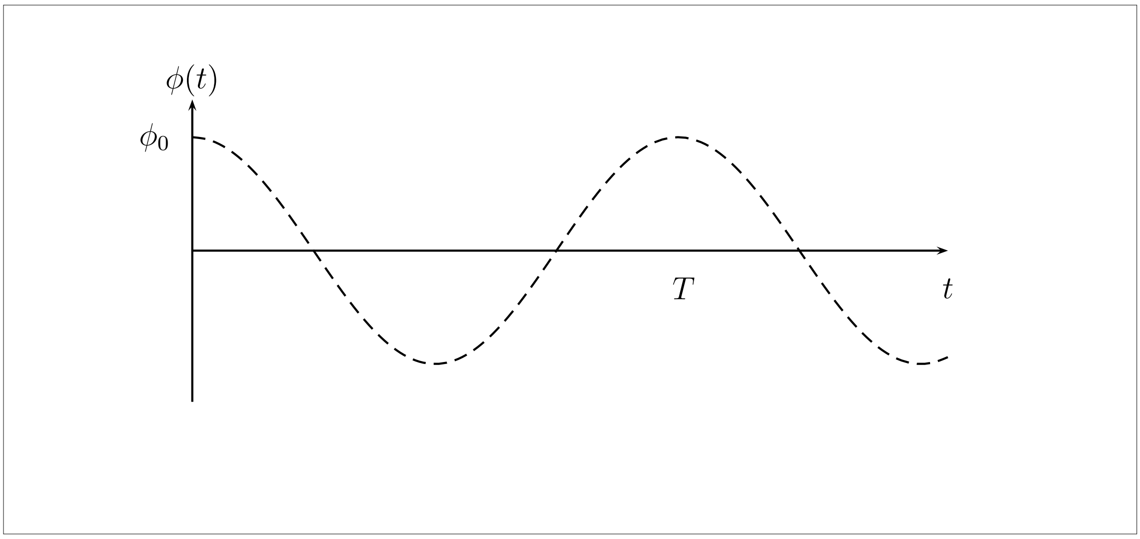
(n) Summarize the results by stating the final solution for the given system.

$$\boxed{\underline{\underline{\phi(t) = \phi_0\cos(\omega t)}}}$$

(o) Calculate the period of the oscillation T.

$$\boxed{T = \tfrac{2\pi}{\omega} = 2\pi\sqrt{\tfrac{12}{13}\tfrac{\ell}{g}}}$$

(p) Draw the time dependence of the solution $\phi(t)$. Make sure to draw the axes, label them and give a scale. Does the function intersect the axes at the right values?


$\phi(t)$
ϕ_0
T
t