**Pre-Semester Physics - Exercises** **Summer 2010**

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**Extra Sheet V**
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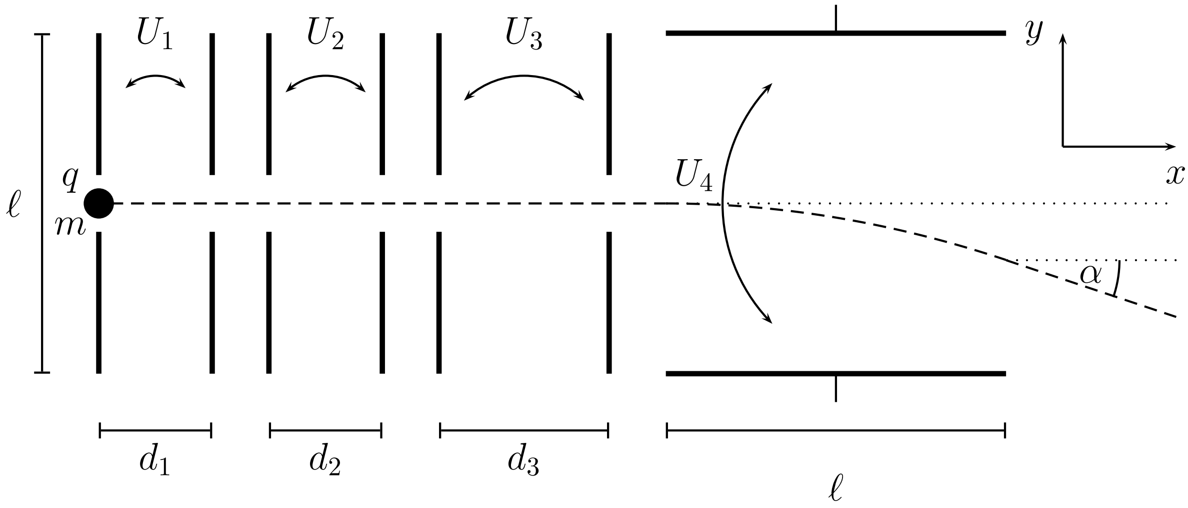
## 1. Exercise:




A point charge $q$ with mass $m$ is passing four capacitors as shown above (where it enters the last capacitor at a distance $\ell/2$ from its plates). While it is at rest before the first capacitor, its velocity after each capacitor is denoted $v_1$, $v_2$, $v_3$ and $v_4$. The plates of the capacitors are all squares with edge length $\ell$ but they are at different distances $d_1 = 2\,\text{m}$, $d_2$, $d_3$ and $\ell$ to each other as well are on different voltages $U_1$, $U_2$, $U_3$ and $U_4$. The goal is to determine the speed and its direction (given by the angle $\alpha$) when the point charge leaves the last capacitor.

(a) Suppose only the electric field $E_1$ inside the first capacitor is given. How can be the voltage drop $U_1$ be obtained?

$$\boxed{U_1 = E_1 d_1}$$

(b) Evaluate it for $E_1 = 140\,\frac{\text{N}}{\text{C}}$.

$$\boxed{U_1 = 280\,\text{V}}$$

(c) Write down the law of energy conservation for the position of the point charge at the left plate and the position of the point charge at the right plate of the first capacitor.

$$\boxed{qU_1 = \frac{1}{2}mv_1^2}$$

(d) Solve this equation to the velocity $v_1$ after the first capacitor.

$$\boxed{v_1 = \sqrt{\frac{2qU_1}{m}}}$$

(e) State the law of energy conservation for the position of the point charge at the left plate and the position of the point charge at the right plate of the second capacitor.

$$\boxed{qU_2 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2}$$

(f) Now write down the law of energy conservation for the position before any capacitor and the position before entering the forth capacitor.

$$\boxed{qU_1 + qU_2 + qU_3 = \frac{1}{2}mv_3^2}$$

(g) Determine from it the velocity before entering the forth capacitor.

$$\boxed{v_3 = \sqrt{\frac{2q(U_1+U_2+U_3)}{m}}}$$

(h) Evaluate this formula for $q = 25\,\mathrm{mC}$, $m = 10\,\mathrm{g}$, $U_2 = 100\,\mathrm{V}$, $U_3 = 150\,\mathrm{V}$.

$$\boxed{v_3 = 52\,\frac{\mathrm{m}}{\mathrm{s}}}$$

(i) Now consider the situation in the last capacitor. What would be the force $F_e$ on the point charge dependent on the applied voltage $U_4$?

$$\boxed{F_e = qE_4 = \frac{qU_4}{\ell}}$$

(j) What kind of motion performs the point charge along the $x$-direction?

☒ uniform motion ☐ motion with constant acceleration ☐ circular motion

(k) What kind of motion performs the body along the $y$-direction?

☐ uniform motion ☒ motion with constant acceleration ☐ circular motion

(l) How does the position in $x$-direction depend on time $t$ for the given problem?

$$\boxed{x(t) = v_3 t}$$

(m) What is the position in $y$-direction in general?

$$\boxed{y(t) = \frac{1}{2}a_y t^2 + v_{y,0}t + y_0}$$

(n) Specify the integration variables for the given problem and for a point of origin at the position of the charge before entering the last capacitor.

$$a_y = -\frac{F_e}{m} = -\frac{qU_4}{m\ell}$$

$$v_{y,0} = 0$$

$$y_0 = 0$$

(o) Determine the velocity in $x$-direction.

$$v_x(t) = v_3$$

(p) How is the velocity in $y$-direction dependent on time?

$$v_y(t) = -\frac{qU_4}{m\ell}t$$

(q) From the equation for the position in $x$-direction, determine the time $t_4$ needed to pass the last capacitor.

$$t_4 = \frac{\ell}{v_3}$$

(r) Check if at this time the point charge is really in the capacitor by evaluating its position in $y$-direction. Use an applied voltage of $U_4 = 480\,\mathrm{V}$.

$$y(t_4) = -\frac{qU_4\ell}{2m{v_3}^2} = -0.23\,\ell > -\frac{\ell}{2} \qquad \checkmark$$

(s) Put the result for the time when it leaves the capacitor into the components of the velocity to obtain their results after passing the last capacitor.

$$v_{x,4} = v_3$$

$$v_{y,4} = -\frac{qU_4}{mv_3}$$

(t) Evaluate them for the given values.

$$v_{x,4} = 52\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$v_{y,4} = -23\,\frac{\mathrm{m}}{\mathrm{s}}$$

(u) Calculate the absolute value of the final velocity.

$$\underline{\underline{v}} = \sqrt{{v_{x,4}}^2 + {v_{y,4}}^2} = \underline{\underline{57\,\frac{\mathrm{m}}{\mathrm{s}}}}$$

(v) Determine the angle $\alpha$ with which the point charge leaves the last capacitor.

$$\underline{\underline{\alpha}} = \arctan\left(\frac{v_{y,4}}{v_{x,4}}\right) = \underline{\underline{24^\circ}}$$