KIT - International Department GmbH

Pre-Semester Physics - Exercises   Summer 2010

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1. Kinematics and Kinetics

(a) What is the (general) $x(t)$ dependence for a linear motion with a constant acceleration?

(b) Give the (general) $x(t)$ dependence for an uniform motion.

(c) At what distance s will a mass point again touch the ground if it is shot from it with a velocity v_0 at an angle α from the horizontal?

(d) What would be the maximum height h of the mass of the last question?

(e) State and explain the content of Newton's laws.

(f) Consider a mass point with mass m on an inclined plane (which has an angle α with respect to the horizontal). Write down the force (due to gravity) acting on the body.

(g) What is the acceleration of the mass in the last question?

(h) Consider two masses m_1, m_2 at the ends of a rope which lies over a pulley (Atwood's machine). Determine the acceleration of the system and the tension force.

(i) How does the acceleration change if the mass m_1 is put on an inclined plane with angle α?

(j) What ratio of the masses is need so that they stay at rest (if again one is put on an inclined plane)?

Solution:

(a) $x(t) = \frac{1}{2}at^2 + v_0 t + x_0$

(b) $x(t) = v_0 t + x_0$

(c) With the impact time T where $y(T) = -\frac{1}{2}gT^2 + v_0 \sin(\alpha) \cdot T = 0$:

$$s = x(T) = v_0 \cos(\alpha) \cdot T = v_0 \cos(\alpha) \cdot 2\frac{v_0 \sin(\alpha)}{g} = \frac{{v_0}^2}{g} \sin(2\alpha)$$

(d) From the symmetry of a parabola:

$$h = y\left(\frac{T}{2}\right) = -\frac{1}{2}g\left(\frac{T}{2}\right)^2 + v_0 \sin(\alpha)\frac{T}{2} = \frac{{v_0}^2}{2g}\sin^2(\alpha)$$

(e) Forces are the cause of a change of a motion : $\vec{F} = 0 \quad \Leftrightarrow \quad \vec{v} = const.$

The mass describes the proportionality : $\vec{F} = m\vec{a}$

Actio = Reactio : $\vec{F}_{12} = -\vec{F}_{21}$

(f) From the vector decomposition of gravity mg as force: $F = mg\sin(\alpha)$

(g) From Newton's second axiom: $a = g\sin(\alpha)$

(h) In writing Newton's second axiom for each mass and solve for the acceleration and the tension force one obtains:

$$\begin{aligned} a &= \frac{m_2 - m_1}{m_1 + m_2} g \\ F_T &= m_1 a + m_1 g = 2\frac{m_1 m_2}{m_1 + m_2} g \end{aligned}$$

(i) Since the acceleration on an inclined plane is reduced by $\sin(\alpha)$:

$$a = \frac{m_2 - m_1 \sin(\alpha)}{m_1 + m_2} g$$

(j) This acceleration vanishes for

$$\frac{m_2}{m_1} = \sin(\alpha)\,.$$

2. Energy and Collisions

(a) What is the time derivative of the energy of a closed system?

(b) At what height h will a mass m stop on an inclined plane when it is moving towards it with velocity v?

(c) What would be the maximum compression of a spring with spring constant D on an inclined plane with angle α if a mass m is falling on it from a height h?

(d) Which conditions (i.e. two equations) have to be fulfilled for a total elastic collision of two particles?

(e) Give in general the conditions (i.e. two equations) for a total inelastic collision of two particles.

(f) What would be the final velocity v' of a mass m_1 after a total elastic collision with a mass m_2?

(g) How fast would be the second mass in the last question if only the first mass moves with velocity v before the collision?

Solution:

(a) Since energy conservation holds: $\dot{E} = 0$

(b) From energy conservation:

$$mgh = \frac{1}{2}mv^2 \qquad \Rightarrow \qquad h = \frac{v^2}{2g}$$

(c) Again energy conservation (for a zero point of the potential energy at the position of the spring) states

$$\begin{aligned} mgh &= \frac{1}{2}Ds^2 - mgs\sin\alpha \\ \Rightarrow \qquad s &= \frac{m}{D}g\sin(\alpha) + \sqrt{\frac{m^2}{D^2}g^2\sin^2(\alpha) + 2\frac{m}{D}gh}\,. \end{aligned}$$

(d) Momentum and energy conservation holds:

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1v_1' + m_2v_2' \\ \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 &= \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2 \end{aligned}$$

(e) Some energy is lost:

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1v_1' + m_2v_2' \\ \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 &= \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2 + Q \end{aligned}$$

(f) Solving the conservation laws gives

$$v' = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}\,.$$

(g) Analog the other velocity can be determined which states for $v_2 = 0$:

$$v_2' = \frac{2m_1v}{m_1 + m_2}$$

3. Rigid body and Oscillations

(a) State the velocity v of a particle moving in a circle with radius r and period T.

(b) What is the moment of inertia of a system composed of a ring with radius r, mass m_r and moment of inertia Θ_r surrounding a point mass m_p with respect to the position of the point mass (i.e. the center of the ring)?

(c) How would it change if the system would be rotated around an axis parallel to the original one, but at the edge of the ring?

(d) Where lies the center of mass of a system composed of an uniform rod of length ℓ with mass m_r and a point mass m at the end of the rod?

(e) Using a conservation law, determine the angular velocity ω of a physical pendulum with moment of inertia Θ and mass m at the vertical position if its center of mass is released at a height h.

(f) Write down the equation of motion for a harmonic oscillator in general and give the general solution.

(g) Give the equations of motion for a simple pendulum and a physical pendulum. In both cases write down the solutions for small angles.

(h) What is the reduced length of a physical pendulum with moment of inertia Θ, mass m and distance ℓ between the center of mass and the axis of rotation?

Solution:

(a) $v = \omega r = \frac{2\pi r}{T}$

(b) Since the total moment of inertia is the sum over its parts:

$$\Theta_{\mathrm{CM}} = \Theta_r + m_p \cdot 0^2 = \Theta_r$$

(c) From the parallel axis theorem the solution reads

$$\Theta_{\mathrm{tot}} = \Theta_{\mathrm{CM}} + (m_r + m_p)r^2 = \Theta_r + (m_r + m_p)r^2 \,.$$

(d) The distance of the center of mass from the point mass along the rod is given by

$$d = \frac{m_r \ell}{2(m_p + m_r)} \,.$$

(e) The solution can be found by energy conservation:

$$mgh = \frac{1}{2}\Theta\omega^2 \qquad \Rightarrow \qquad \omega = \sqrt{\frac{2mgh}{\Theta}}$$

(f) $\ddot{x}(t) + \omega^2 x(t) = 0 \qquad \Leftrightarrow \qquad x(t) = \mathcal{A}\cos(\omega t + \phi)$

(g)

$$\ddot{\alpha}(t) + \frac{g}{\ell}\sin(\alpha(t)) = 0 \qquad \rightarrow \qquad \alpha(t) = \mathcal{A}\cos\left(\sqrt{\frac{g}{\ell}}t + \phi\right)$$

$$\ddot{\alpha}(t) + \frac{mg\ell}{\Theta}\sin(\alpha(t)) = 0 \qquad \rightarrow \qquad \alpha(t) = \mathcal{A}\cos\left(\sqrt{\frac{mg\ell}{\Theta}}t + \phi\right)$$

(h) In comparing both formulas the reduced length is obtained:

$$\ell_r = \frac{\Theta}{m\ell}$$