KIT - International Department GmbH

Pre-Semester Physics - Exercises Summer 2010

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1. Electrostatics

(a) What is the electric field E of a point charge Q at a distance r?

(b) Write down the electric field E inside a capacitor with parallel plates of area A on which a charge Q is placed.

(c) In which direction does it point (if the upper plate is positively charged)?

(d) What would be the field outside?

(e) State the electric force $\vec{F}$ on a charge q in an electric field $\vec{E}$.

(f) How is the electric field E related to the voltage U and the distance d of the plates of a parallel plate capacitor?

(g) What energy W does a charge q with mass m gain when passing a parallel plate capacitor charged by a voltage U?

(h) State the voltage U needed to accelerated the point charge to a speed v after the capacitor if it is emitted from one of its plates with negligible speed.

(i) What kind of motion does a charge q with mass m if it enters a parallel plate capacitor perpendicular to the electric field E? Give explicit the $\vec{r}(t)$ and $\vec{v}(t)$ dependence and the geometrical path $y(x)$ for that kind of motion if it enters at the point of origin with velocity v_0 parallel to the plates.

(j) Write down the absolute value of the velocity v which the charge posses after leaving the capacitor.

(k) How can the angle α be obtained, describing the direction the charge leaves the capacitor?

(l) State the voltage U which lies on a capacitor with capacitance C charged by Q.

(m) How can the dielectric constant ϵ_r of a material be determined if the capacitance C of a parallel plate capacitor with edge length ℓ and distance d between the plates is known where the material is placed between its plates?

Solution:

(a) $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$

(b) $E = \frac{Q}{\epsilon_0 A}$

(c) The electric field pointing downwards from the positively to the negatively charged plate.

(d) $E = 0$

(e) $\vec{F_e} = q\vec{E}$

(f) $E = \frac{U}{d}$

(g) $W = qU$

(h) $U = \frac{mv^2}{2q}$

(i) It performs a uniform motion parallel and a motion with constant acceleration perpendicular to the plates of the capacitor:

$$\vec{v}(t) = \begin{pmatrix} v_0 \\ \frac{qE}{m}t \\ 0 \end{pmatrix}$$

$$\vec{r}(t) = \begin{pmatrix} v_0 t \\ \frac{1}{2}\frac{qE}{m}t^2 \\ 0 \end{pmatrix}$$

$$y(x) = \frac{qE}{2mv_0^2}x^2$$

(j) $v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{{v_0}^2 + \frac{q^2E^2\ell^2}{m^2{v_0}^2}}$

(k) $\tan(\alpha) = \frac{v_y}{v_x} \quad \Rightarrow \quad \alpha = \arctan\left(\frac{qE\ell}{m{v_0}^2}\right)$

(l) $U = \frac{Q}{C}$

(m) $\epsilon_r = \frac{Cd}{\epsilon_0 \ell^2}$

2. Electromagnetism

(a) Explain the content of Maxwell's equations.

(b) Write down the magnetic field B of a wire through which a current I runs depending to the distance r to it.

(c) State the magnetic field B inside a coil with length ℓ, base area A and turns N through which runs a current I.

(d) In which direction does it point (if the current runs anti-clockwise)?

(e) What would be the magnetic field outside?

(f) Write down the magnetic part of the Lorentz-Force, i.e. the force of a magnetic field $\vec{B}$ on a charge q, that moves with velocity $\vec{v}$.

(g) How can one calculate the force F of a magnetic field B on a wire of length ℓ carrying a current I?

(h) What kind of motion describes a point charge q with mass m in homogeneous magnetic field B if its velocity v would be perpendicular to the magnetic field? State its time and length characteristics T, r.

(i) How would the motion change if the velocity is not perpendicular to the magnetic field?

(j) Explain the function of a velocity selector.

(k) State the definition of the magnetic flux Φ.

(l) What is the induced voltage U of a quadratic frame with edge length ℓ rotating in a homogeneous magnetic field B for an initial maximum value?

(m) Write down the definition of the self-inductance $\mathcal{L}$.

(n) How is the self-inductance $\mathcal{L}$ of a filled coil related to its geometric quantities?

(o) What kind of energies are converted in a generator, an electric motor and a transformer? State the energy conservation laws in terms of powers for these cases.

Solution:

(a) Maxwell's equations:

- Gauss' law: $\oint_S \vec{E} \mathrm{d}\vec{A} = \frac{Q_{\text{inside}}}{\epsilon_0}$

  If the summation of the electric field around a closed surface is not vanishing, there have to be charges inside it generating the electric field.

- Gauss' law for magnetism: $\oint_S \vec{B} \mathrm{d}\vec{A} = 0$

  The magnetic field is not generated by some kind of (resting) charges. (The magnetic field-lines are closed.)

- Faraday's law of induction: $\oint_C \vec{E} \mathrm{d}\vec{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{B} \mathrm{d}\vec{A}$

  If the magnetic field in an area (or more precise the magnetic flux) is changing with time, it will cause an voltage around the edge of such a surface.

- Ampere's law: $\oint_C \vec{B} \mathrm{d}\vec{\ell} = \mu_0 \int_S \vec{j} \mathrm{d}\vec{A} + \frac{\mathrm{d}}{\mathrm{d}t} \int_S \vec{E} \mathrm{d}\vec{A}$

  A magnetic field is generated around a current.

(b) $B = \frac{\mu_0 I}{2\pi r}$

(c) $B = \mu_0 \frac{N}{\ell} I$

(d) If one orientates the coil so that the current runs anti-clockwise, the magnetic field will come towards one.

(e) $B = 0$

(f) $\vec{F}_{\text{L,mag}} = q\vec{v} \times \vec{B}$

(g) $F = I\ell B_\perp$

(h) It would describe a circular motion with radius $r = \frac{mv}{qB}$ and period $T = \frac{2\pi m}{qB}$.

(i) Since the parallel component would shift the circles it becomes a helical motion.

(j) When putting an electrostatic force opposite to a magnetic force only one velocity $v = \frac{E}{B}$ would not be deflected.

(k) $\Phi = \int \vec{B} \mathrm{d}\vec{A}$

(l) $U = -\dot{\Phi} = -B\ell^2\omega\cos(\omega t)$

(m) $\mathcal{L} = \frac{|U|}{|\dot{I}|}$

(n) $\mathcal{L} = \mu_0 \mu_r \frac{NA}{\ell}$

(o) Generator: rotation energy into electric energy (in an AC-circuit) $M\omega = UI$
Electric motor: electric energy into rotation energy $UI = M\omega$
Transformer: electric energy of AC-circuits at different rms-voltages and rms-currents $U_1 I_1 = U_2 I_2$

3. Circuits

(a) Explain the content of Kirchhoff rules.

(b) How is the resistance R of a wire related to its geometric quantities?

(c) State Ohm's law.

(d) What is the power P is dissipated in a resistor R if it lies at a voltage U?

(e) How can the equivalent resistance R_{tot} of two parallel resistors R_1 and R_2 be calculated?

(f) What is the equivalent resistance of two serial connected resistors?

(g) Calculate the current I which would flow between two connected points which lie on the same potential.

(h) Sketch the $I(t)$ dependence of the opening and closing (discharging and charging) of a $R\mathcal{L}$ circuit.

(i) How is the charge depending on time if the capacitor in a RC circuit is charged or discharged?

(j) After what time t posses the current I half its initial value I_0 in the chase of discharging a capacitor in an RC circuit?

(k) What is period of a $RC\mathcal{L}$-circuit?

Solution:

(a) The directed sum of the current in each branch connection and the directed sum of the voltage in each loop vanishes: $\sum_k I_k = 0,\ \sum_k U_k = 0$.

(b) $R = \rho_s \frac{\ell}{A}$

(c) $R = \frac{U}{I}$

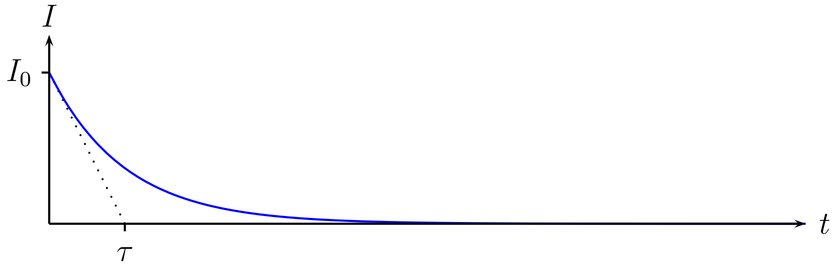
(d) $P = UI = \frac{U^2}{R}$

(e) $\frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2}$

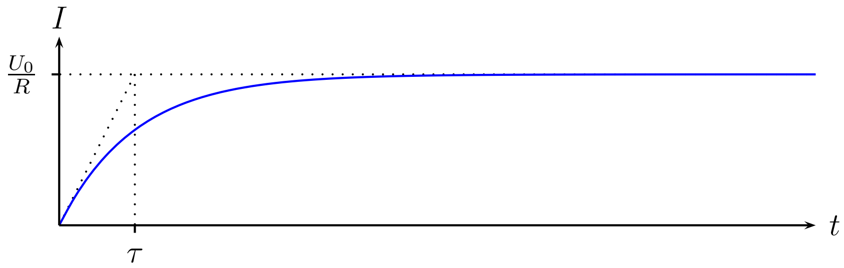
(f) $R_{\text{tot}} = R_1 + R_2$

(g) $I = 0$ since $U = 0$

(h) Opening / Discharging: Exponential decay of the initial current I_0:



Closing / Charging: Negative exponential decay from zero to the maximal current $\frac{U_0}{R}$.



The time constant is given by $\tau = \frac{\mathcal{L}}{R}$

(i) Opening / Discharging: $Q(t) = Q_0 \mathrm{e}^{-t/\tau}$ where the time constant is $\tau = RC$

Closing / Charging: $Q(t) = CU_0 \left(1 - \mathrm{e}^{-t/\tau}\right)$

(j) $t = RC\ln(2)$

(k) $T = \frac{2\pi}{\sqrt{\frac{1}{\mathcal{L}C} - \frac{R^2}{4\mathcal{L}^2}}}$