KIT - International Department GmbH

Pre-Semester Physics - Exercises Summer 2010

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Test-Exam
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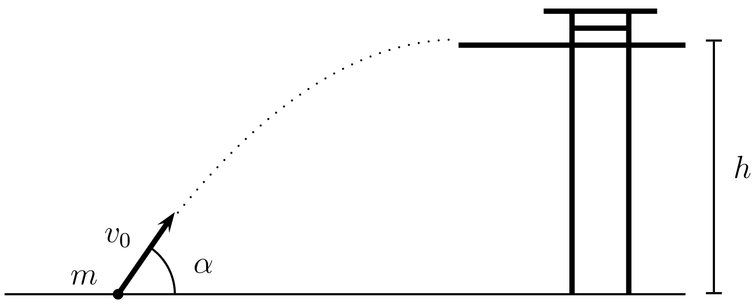
1. Exercise (10 P)



A point mass is shot with an initial velocity of $v_0 = 30\,\frac{\text{m}}{\text{s}}$ at $\alpha = 45°$ from the horizontal towards a platform at height h. Along the vertical gravity is acting with $g = 10\,\frac{\text{m}}{\text{s}^2}$.

(a) Write down the path of the mass either in vector form (i.e. $\vec{r}(t)$) or in component form (i.e. $x(t)$ and $y(t)$). **2 P**

(b) Determine the maximum height h of the mass point and the time T when it reaches it. **2 P**

(c) State the angle α_{max} when the height is maximal. **2 P**

Solution:

(a) Along the x-direction a uniform motion is taking place, while along the y direction a motion with constant acceleration is been performed:

$$\underline{\underline{\vec{r}(t)}} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} v_0 \cos(\alpha) \cdot t \\ -\frac{1}{2}gt^2 + v_0 \sin(\alpha) \cdot t \end{pmatrix} = \underline{\underline{\begin{pmatrix} 21\,\frac{\text{m}}{\text{s}} \cdot t \\ -5\,\frac{\text{m}}{\text{s}^2} \cdot t^2 + 21\,\frac{\text{m}}{\text{s}} \cdot t \end{pmatrix}}}$$

with the point of origin at the initial position of the point mass.

(b) At the maximum height the y-component of the velocity vanishes. Therefore the point mass would be at its maximum height at

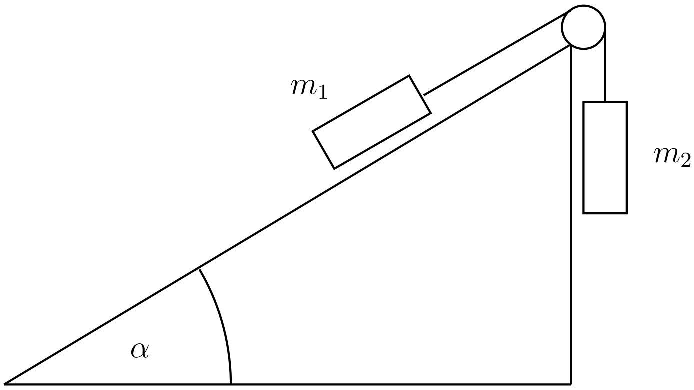
$$v_y(t) = -gt + v_0 \sin(\alpha) \qquad \text{with} \qquad v_y(T) \stackrel{!}{=} 0$$

$$\Rightarrow \qquad \underline{\underline{T}} = \frac{v_0}{g} \sin(\alpha) = \underline{\underline{2.1\,\text{s}}}$$

Thus, the height is given by the y-coordinate:

$$\underline{\underline{h}} = y(T) = -\frac{1}{2}gT^2 + v_0 \sin(\alpha) T = \frac{{v_0}^2}{2g} \sin^2(\alpha) = \underline{\underline{22.5\,\text{m}}}$$

(c) From the last formula $\sin^2(\alpha)$ should be maximal, i.e. $\sin(\alpha)$ should have a maxima, in order to gain the maximum height. This is the case for $\underline{\underline{\alpha = 90°}}$.



Now consider two masses $m_1 = 20\,\text{kg}$ and $m_2 = 30\,\text{kg}$ which are connected by a rope running over a pulley. Both masses are exposed to the gravitational force ($g = 10\,\frac{\text{m}}{\text{s}^2}$). However, the mass m_1 lies on an inclined plane with angle $\alpha = 30°$ (with respect to the horizontal). Neglect friction!

(d) Calculate the acceleration a of mass m_2. **2 P**

(e) State the tension force F_T in the rope. **2 P**

Solution:

(d) Newton's second axiom for both masses reads

$$\begin{aligned} m_1 a_1 &= -m_1 g \sin(\alpha) + F_T \\ m_2 a_2 &= \quad m_2 g \qquad\quad - F_T \end{aligned}$$

where the forces were measured in the direction to the pulley for mass m_1 and away from the pulley for mass m_2. If the mass m_2 moves along the vertical, the mass m_1 would move due to the rope the same distance along the inclined plane. That means

$$x_1 = x_2 \qquad \Rightarrow \qquad \ddot{x}_1 = \ddot{x}_2 \qquad \text{or} \qquad a_1 = a_2 = a$$

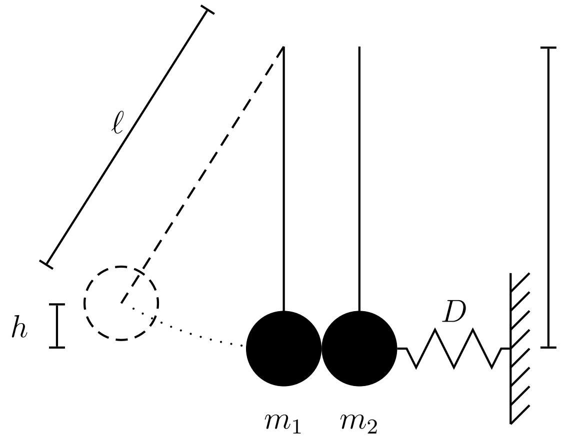
if measured from the initial positions. Therefore we can eliminate the tension force F_T in the equations of motion:

$$\begin{aligned} F_T &= m_1 a + m_1 g \sin(\alpha) \\ m_2 a &= m_2 g - m_1 g \sin(\alpha) - m_1 a \qquad \Rightarrow \qquad \underline{\underline{a}} = \frac{m_2 - m_1 \sin(\alpha)}{m_1 + m_2} g = \underline{\underline{4\,\frac{\text{m}}{\text{s}}}} \end{aligned}$$

(e) As determined in the first part the tension force reads

$$\underline{\underline{F_T}} = m_1\,(a + g \sin(\alpha)) = \underline{\underline{180\,\text{N}}}\,.$$

2. Exercise **(10 P)**



Two masses $m_1 = 2\,\mathrm{kg}$ and $m_2 = 3\,\mathrm{kg}$ are hanging on parallel ropes with length $\ell = 40\,\mathrm{m}$, so that at the equilibrium position they touch each other. At $t = 0$ the first mass m_1 is put at a height $h = 20\,\mathrm{m}$ while the second mass at the equilibrium position is put at a spring with spring constant $D = 300\,\frac{\mathrm{N}}{\mathrm{m}}$. The mass m_1 is then released in order to collide elastically with the mass m_2. Neglect friction!

(a) Calculate the angle α between the rope of the first mass m_1 and the vertical at $t = 0$. **2 P**

(b) Determine the velocity v of the first mass before the collision. **2 P**

(c) State the velocity of both masses v_1' and v_2' after a total elastic collision where the influence of the spring can be neglected. **4 P**

(d) Find the maximum compression s_2 of the spring. **2 P**

Solution:

(a) From geometry the height h can be derived, determining the angle:

$$h = \ell(1 - \cos(\alpha)) \qquad \Rightarrow \qquad \underline{\underline{\alpha}} = \arccos\left(1 - \frac{h}{\ell}\right) = \underline{\underline{60^\circ}}$$

(b) By energy conservation the velocity can be calculated:

$$\frac{1}{2}m_1 v^2 = m_1 g h \qquad \Rightarrow \qquad \underline{\underline{v}} = \sqrt{2gh} = \underline{\underline{20\,\frac{\mathrm{m}}{\mathrm{s}}}}$$

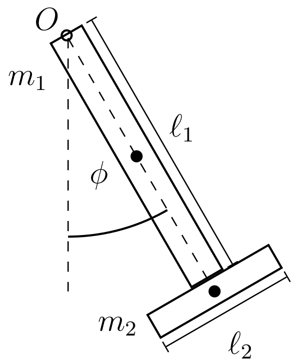
(c) Since it is a total elastic collision momentum and energy conservation holds. From them the velocities after the collision can be found:

$$\underline{\underline{v_1'}} = \frac{m_1 - m_2}{m_1 + m_2} v_1 = \underline{\underline{-4\,\frac{\mathrm{m}}{\mathrm{s}}}} \qquad \underline{\underline{v_2'}} = \frac{2m_1}{m_1 + m_2} v_1 = \underline{\underline{16\,\frac{\mathrm{m}}{\mathrm{s}}}}$$

(d) Again energy conservation - this time with the potential energy of a spring - leads to

$$\frac{1}{2} D {s_2}^2 = \frac{1}{2} m_2 {v_2'}^2 \qquad \Rightarrow \qquad \underline{\underline{s}} = \sqrt{\frac{m_2}{D}} v_2' = \underline{\underline{1.6\,\mathrm{m}}}\ .$$

3. Exercise **(10 P)**



A hammer is hanged at O, the end of one of its rods. It consists of two perpendicular rods with masses $m_1 = 49 \cdot m$ and $m_2 = 80 \cdot m$, and lengths $\ell_1 = \ell$, $\ell_2 = \ell/2$ which are connected at the center of the second rod. Neglect the short edge lengths of the rods as well as friction, but consider gravity acting along the vertical. Initially, the hammer is at rest and placed at an angle $\phi(0) = \phi_0$ with respect to its equilibrium position.

(a) What would be the equation of motion and the period $\tilde{T}$ for small angles if the mass m_1 as well as the length ℓ_2 of the second rod could be neglected (but m_2 and ℓ_1 stay finite)? **2 P**

(b) Show that the total moment of inertia is given by

$$\Theta_{\text{tot}} = 98\, m\, \ell^2 \,. \qquad \textbf{2 P}$$

(c) State the moment of torque M as function of the angle ϕ. **1 P**

(d) Write down the equation of motion for the system and give the general solution $\phi(t)$ for small angles ϕ. **2 P**

(e) What is the period T of the oscillation? **1 P**

(f) Draw the time dependence of the solution $\phi(t)$ and mark where the initial angle ϕ_0 and the period T can be found in the diagram. **1 P**

(g) Sketch how the diagram would change if the hammer is moving through a Newtonian fluid with a small friction parameter. **1 P**

Hint: The moment of inertia for a long, uniform rod of mass m_r and length L with respect to an axis of rotation through the center of mass is given by

$$\Theta_{\text{CM}} = \frac{1}{12} m_r L^2 \,.$$

This need not to be proven.

Solution:

(a) It would then represent a mathematical pendulum with the following equation of motion and period (for small angles):

$$\underline{\underline{\ddot{\phi} = -\frac{g}{\ell_1}\sin(\phi)}} \approx -\frac{g}{\ell_1}\phi = -\omega^2\phi, \qquad \underline{\underline{\tilde{T}}} = \frac{2\pi}{\omega} = \underline{\underline{2\pi\sqrt{\frac{\ell_1}{g}}}}$$

(b) By the parallel axis theorem the moment of inertia is given by

$$\underline{\underline{\Theta}} = \frac{1}{12}m_1\ell^2 + m_1\frac{\ell^2}{4} + \frac{1}{12}m_2\ell^2 + m_2\ell^2 = \frac{16m_1 + 49m_2}{48}\ell^2 = \underline{\underline{98m\ell^2}}\,.$$

(c) The moment of torque is given by the total mass and the coordinate of the center of mass:

$$\underline{\underline{M}} = (m_1 + m_2)\cdot\frac{m_1\frac{\ell}{2} + m_2\ell}{m_1 + m_2}g\sin(\phi) = \underline{\underline{\frac{209}{2}m\ell g\sin(\phi)}}$$

(d) The equation of motion reads

$$\underline{\underline{\ddot{\phi}}} = -\frac{209}{2\Theta}m\ell g\sin(\phi) = \underline{\underline{-\frac{209\,g}{196\,\ell}\sin(\phi)}} \approx -\underbrace{\frac{209\,g}{196\,\ell}}_{\omega^2}\cdot\phi$$

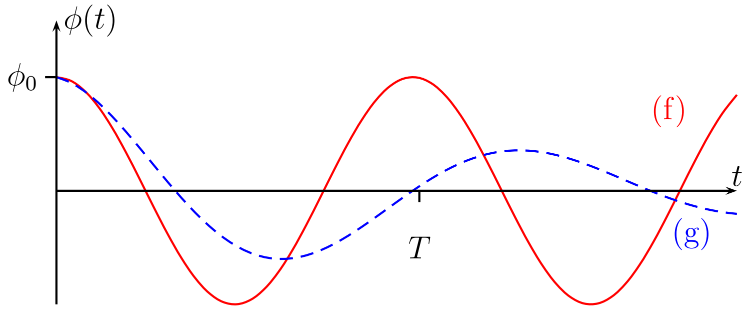
with the general solution

$$\underline{\underline{\phi = \mathcal{A}\cos(\omega t + \tilde{\phi})}}\,.$$

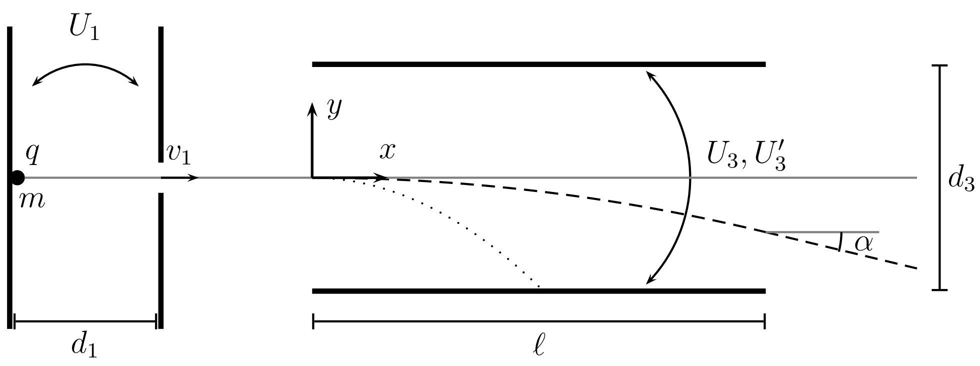
(e)

$$\underline{\underline{T}} = \frac{2\pi}{\omega} = \underline{\underline{2\pi\sqrt{\frac{196\,\ell}{209\,g}}}}$$

(f),(g)



4. Exercise **(10 P)**



A point charge with $q = 40\,\mathrm{mC}$ and mass $m = 20\,\mathrm{g}$ is emitted and accelerated inside a parallel plate capacitor to the velocity $v_1 = 40\,\frac{\mathrm{m}}{\mathrm{s}}$. The capacitor is lying at the potential difference U_1 and having a separation distance of its plate d_1. The point charge then enters at the center a parallel plate capacitor with the velocity parallel to the plates. The quadratic plates of the second parallel plate capacitor are separated by a distance $d_1 = 30\,\mathrm{m}$ and are of length $\ell = 20\,\mathrm{m}$. Neglect the gravitational force.

(a) State the electric fields and its directions in the first, the second and between both capacitors for the path shown above. **3 P**

(b) Using a conservation law, calculate the voltage U_1 needed so that the point charge leaves the first capacitor with a velocity of v_1. **2 P**

(c) State the absolute speed v_3 and the angle α, the point charge leaves the capacitor with if a voltage of $U_3 = 600\,\mathrm{V}$ would be applied to the second capacitor. **3 P**

(d) Determine the potential difference U_3' which is needed such that the point charge hits the parallel plate capacitor in the middle of the plate (see dotted curve in the figure). **2 P**

Solution:

(a) In the first capacitor the electric field is given by $\underline{\underline{E_1 = \frac{U_1}{d_1}}}$ and is pointing $\underline{\underline{\text{to the right}}}$. Between the capacitors the field vanishes $\underline{\underline{E_2 = 0}}$. In the second capacitor the field is found to be $\underline{\underline{E_3 = \frac{U_3}{d_3}}}$ and is pointing $\underline{\underline{\text{downwards}}}$.

(b) From energy conservation the voltage needed can be derived:

$$qU_1 = \frac{1}{2}mv_1^2 \qquad \Rightarrow \qquad \underline{\underline{U_1}} = \frac{m{v_1}^2}{2q} = \underline{\underline{400\,\mathrm{V}}}$$

(c) Since the point charge performs a motion with constant acceleration along the y-direction and a uniform motion along the x-direction the velocity can be calculated:

$$\underline{\underline{v_3}} = \sqrt{{v_1}^2 + \frac{q^2{U_3}^2\ell^2}{m^2{d_3}^2{v_1}^2}} = \underline{\underline{45\,\tfrac{\mathrm{m}}{\mathrm{s}}}}$$

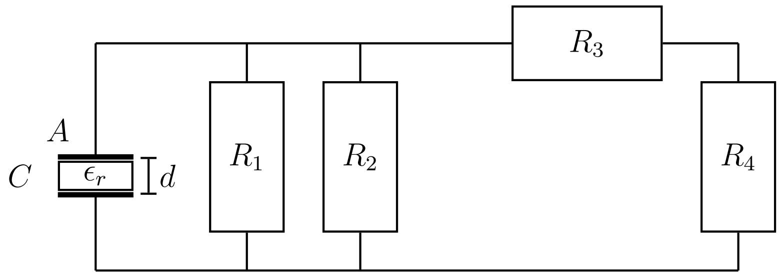
$$\underline{\underline{\alpha}} = \arctan\left(\frac{qU_3\ell}{md_3{v_1}^2}\right) = \underline{\underline{26.6°}}$$

$$\left(\qquad y\left(\frac{\ell}{v_1}\right) = \frac{qU_3\ell^2}{2md_3{v_1}^2} = 5\,\mathrm{m} < 15\,\mathrm{m} = \frac{d_3}{2} \qquad \right)$$

(d) The voltage needed can by determined by a kinetic consideration:

$$\frac{d_3}{2} = y\left(\frac{\ell}{2v_1}\right) = \frac{qU_3'}{2md_3} \cdot \frac{\ell^2}{4{v_1}^2} \qquad \Rightarrow \qquad \underline{\underline{U_3'}} = \frac{4m{d_3}^2{v_1}^2}{q\ell^2} = \underline{\underline{7.2\,\mathrm{kV}}}$$

5. Exercise **(10 P)**



Consider a capacitor with capacitance of $C = 10\,\mu\text{F}$ and an area of its parallel plates $A = 10\,\text{m}^2$. The plates are separated by a distance $d = 177\,\mu\text{m}$ and between them a dielectric material is present, too. Initially, the charge $Q_0 = 1.0\,\text{mC}$ is placed on the capacitor. It is connected to a network of four resistors with resistances $R_1 = 2\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 5\,\Omega$, $R_4 = 7\,\Omega$ as pictured in the figure.

(a) Determine the electric permittivity ϵ_r of the material inside the capacitor. **1 P**

(b) State the voltage U_0 which drops initially at the capacitor. **1 P**

(c) Calculate the resistance R_{tot} of the whole circuit. **3 P**

(d) What is the initial current I_0? **1 P**

(e) Find the current I_1 which flows initially through the resistor R_1. **2 P**

(f) When does the current through the resistor R_1 reach the value $I_1/4$? **2 P**

Solution:

(a) The relative permittivity is given by

$$C = \epsilon_0 \epsilon_r \frac{A}{d} \qquad \Rightarrow \qquad \underline{\underline{\epsilon_r}} = \frac{Cd}{\epsilon_0 A} = \underline{\underline{20}}\ .$$

(b) The initial voltage is determined by the definition of the capacitance:

$$C = \frac{Q_0}{U_0} \qquad \Rightarrow \qquad \underline{\underline{U_0}} = \frac{Q_0}{C} = \underline{\underline{100\,\mathrm{V}}}$$

(c) The total resistance can be calculated by

$$\underline{\underline{R_{\mathrm{tot}}}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3+R_4}} = \underline{\underline{\frac{12}{11}\,\Omega}}\ .$$

(d) From it the initial current can by determined:

$$R_{\mathrm{tot}} = \frac{U_0}{I_0} \qquad \Rightarrow \qquad \underline{\underline{I_0}} = \frac{U_0}{R_{\mathrm{tot}}} = \underline{\underline{92\,\mathrm{A}}}$$

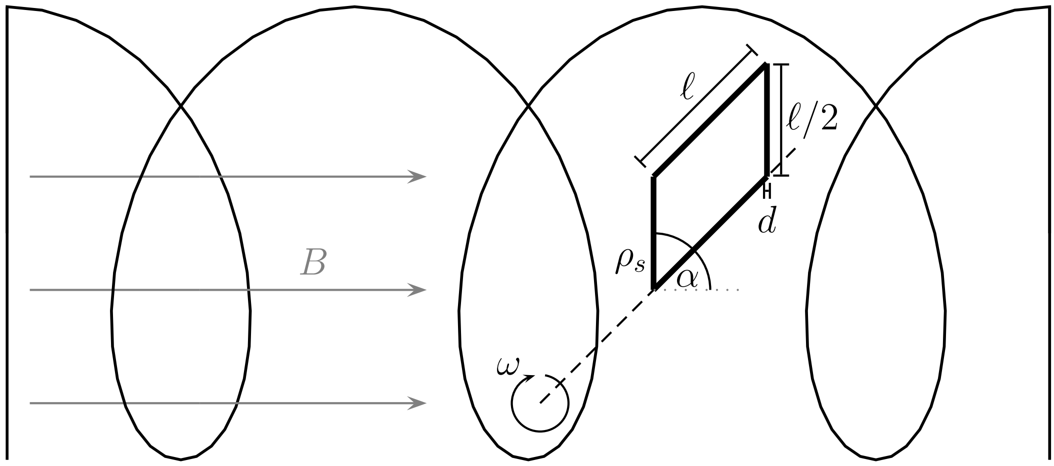
(e) From Ohm's law the current through the resistor R_1 is given by

$$R_1 = \frac{U_0}{I_1} \qquad \Rightarrow \qquad \underline{\underline{I_1}} = \frac{U_0}{R_1} = \underline{\underline{50\,\mathrm{A}}}\ .$$

(f) The current will be decreased by 25% at

$$\frac{I_1}{4} = I_1 \mathrm{e}^{t/R_{\mathrm{tot}}C} \qquad \Rightarrow \qquad \underline{\underline{t}} = R_{\mathrm{tot}} C \ln(4) = \underline{\underline{15\,\mu\mathrm{s}}}\ .$$

6. Exercise **(10 P)**



Inside a coil, where a current I_c produces a magnetic field B, a metal frame is rotated with constant angular velocity ω and an axis along one of its edges. The edges of the quadratic frame consist of a wire with length ℓ and $\ell/2$, a cross section area with small diameter d, and a specific resistance of ρ_s. Initially, the frame is parallel to the turns of the coil.

(a) Does the current I_c flows clockwise or anti-clockwise when looking in the direction of the magnetic field? **1 P**

(b) What is the total resistance R of the metal frame? **1 P**

(c) Determine the induced voltage $U_{\text{ind}}(t)$. **2 P**

(d) Find the current $I(t)$ which runs through the frame in terms of R, B, ℓ and ω. **1 P**

(e) What force $F(t)$ acts on the rotating edge of the metal frame parallel to the rotation axis? **2 P**

(f) Calculate the moment of torque $M(t)$ necessary to keep the frame at constant angular velocity. **1 P**

(g) Compare the power $P_{\text{el}}(t)$ dissipated in the frame to the one $P_{\text{mech}}(t)$ provided by the moment of torque. **2 P**

Solution:

(a) By the right-hand rule the current flows $\underline{\underline{\text{clockwise}}}$.

(b) The total resistance is given by

$$\underline{\underline{R}} = \rho_s \frac{2\ell + 2\frac{\ell}{2}}{\pi \frac{d^2}{4}} = \underline{\underline{\frac{12\ell\rho_s}{\pi d^2}}} .$$

(c) From the magnetic flux the induction voltage is determined:

$$\Phi = B\frac{\ell^2}{2}\cos(\omega t) \qquad \Rightarrow \qquad \underline{\underline{U_{\text{ind}}}} = -\dot{\Phi} = \underline{\underline{B\frac{\ell^2}{2}\omega\sin(\omega t)}}$$

where the direction of the normal vector of the area was defined parallel to the magnetic field, so that the flux would be maximal initially.

(d) The current can by found by Ohm's law:

$$\underline{\underline{I}} = \frac{U_{\text{ind}}}{R} = \underline{\underline{\frac{B\ell^2\omega}{2R}\sin(\omega t)}}$$

This current would flow clockwise in the first quarter of the rotation in order to increase the local magnetic field. Therefore it counteracts the reduction of the magnetic flux in the frame (Lenz's law).

(e) The force on the only rotating wire which is always perpendicular to the magnetic field can be calculated by

$$\underline{\underline{F}} = BI\ell = \underline{\underline{\frac{B^2\ell^3\omega}{2R}\sin(\omega t)}}$$

and is pointing upwards in the first quarter of the rotation (according to the right-hand rule).

(f) The moment of torque is given by

$$\underline{\underline{M}} = \frac{\ell}{2}\cdot F\cdot\sin\omega t = \underline{\underline{\frac{B^2\ell^4\omega}{4R}\sin^2(\omega t)}}$$

and tries to slow down the rotation (Lenz's law or right-hand rule). This moment of torque has therefore be counteracted by the applied one in order to get a resulting vanishing moment of torque ($M + M_{\text{applied}} = 0$). Such a moment of torque is needed in order to satisfy the equation of motion ($\Theta\dot{\omega} = 0$) for a motion with constant angular velocity. The different signs of the applied moment of torque and the one stated would symbolize the difference between power drain and the power provided in the energetic consideration below.

(g) Both powers are equal according to energy conservation:

$$\underline{\underline{P_{\text{el}}}} = U_{\text{ind}}I = \frac{B^2\ell^4\omega^2}{4R}\sin^2(\omega t) = M\omega = \underline{\underline{P_{\text{mech}}}}$$