Karlsruhe Institute of Technology - International Department GmbH

Pre-Semester 2010 - Physics Course - Extra Tutorial

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1. Spring

A point mass attached to a (massless) spring with force constant D is initially displaced by x_0 from its equilibrium position (the spring is, say, stretched). It is released at time $t = 0$.

(a) State the equation of motion (for $x(t)$) and solve it.

(b) At which time t_1 does the oscillator (the spring with mass point) return to its initial state the first time? How would you name such a time?

(c) What is the velocity $v(t)$ at time t?

(d) What are the kinetic and potential energies $E_{\mathrm{kin}}(t)$ and $E_{\mathrm{pot}}(t)$ at time t? Compute their sum E_{tot}.

2. Weakly Damped Harmonic Oscillator

We now assume that the oscillator from the previous exercise moves in a liquid and experiences Stokes friction with coefficient γ. We assume that this damping is weak, $\gamma^2 < 4Dm$.

(a) State the new equation of motion (again for $x(t)$).

(b) Consider the function

$$x(t) = Ae^{-\lambda t}\cos(\omega t + \phi) \quad \text{with} \quad \lambda = \frac{\gamma}{2m} \quad \text{and} \quad \omega = \sqrt{\frac{D}{m} - \frac{\gamma^2}{4m^2}}.$$

Compute its derivatives $\dot{x}(t)$, $\ddot{x}(t)$ and show that it solves the equation of motion.

3. Physical Pendulum

At time $t = 0$ a physical pendulum with moment of inertia $\Theta = 0.22\,\mathrm{kg\,m^2}$ and mass $m = 500\,\mathrm{g}$ makes an angle $\alpha_0 = 7°$ (more precisely: the line connecting rotation axis and center of mass makes this angle) with the vertical and has an angular velocity $\omega_0 > 5.75°/\mathrm{s}$ (it is moving counterclockwise). Its rotation axis and center of mass have a distance $l = 10\,\mathrm{cm}$.

(a) State the equation of motion (for $\alpha(t)$) and approximate it for small angles α.

(b) Solve the equation of motion by using the general ansatz and the *two* initial conditions.

(c) What is the amplitude of the oscillation (in degrees)? What is the period of oscillation?

(d) At which time t_1 will the pendulum reach maximal deflection (maximal α) for the first time? At which time t_2 will it do so for the second time (this time deflection will be negative)?

(e) Assuming that the pendulum has been oscillating for quite some while (before $t = 0$), at which time t_0 was the angle $\alpha = 0$ the last time before $t = 0$?

(f) Sketch the solution.

4. Where's the Angle?

A point mass m is performing a circular motion in the two dimensional $x - y$-plane around the origin O. It moves counterclockwise with constant angular velocity ω along a circle with radius A.

(a) What is the angle $\alpha(t)$ at time t between the line connecting O with the point mass and the x-axis if for $t = 0$ this angle is ϕ?

(b) What are the coordinates $x(t)$ and $y(t)$ of the point mass at time t?

(c) Do you remember how the point mass's speed v is related to the given quantities?

(d) Just to be sure, check that $v^2 = (\dot{x}(t))^2 + (\dot{y}(t))^2$.

(e) Give the force F_{cp} which acts on the point mass. What is the component $F_{\mathrm{cp},x}$ of this force in x-direction? Express it in terms of x (get rid of α or t!).

(f) Although it may seem pointless, since we already know the solution: Using Newton's second law $m\ddot{x} = F_{\mathrm{cp},x}$ in x-direction, state the equation of motion for $x(t)$.