Pre-Semester 2010 - Physics Course - Extra Tutorial

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Solution 1
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1. Units

(a) Conversion between degree and radian – fill in the blanks:

$^\circ$	360	90	60	180	45	30	270
rad	2π	$\frac{\pi}{2}$	$\frac{\pi}{3}$	π	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{3}{2}\pi$

(b) Referring to Newton's second axiom, describe in words how 1 N [Newton] is related to the base units kg, m, and s.

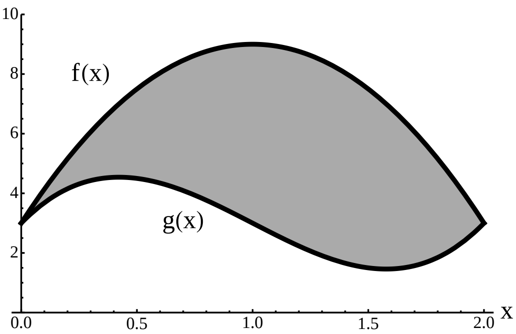
Answer: 1 Newton is the force which is required to accelerate a body of mass 1 kg by 1m/s^2.

2. Differentiation and Integration

(a) Let $F(x)$ denote some antiderivative of $f(x)$: $F'(x) = f(x)$. Fill in the blanks:

$f(x)$	$\sin x$	$\cos x$	$\log x$	$9\ e^{3x}$	$7x^6 + 6x^2$
$f'(x)$	$\cos x$	$-\sin x$	$1/x$	$27\ e^{3x}$	$42x^5 + 8x$
$F(x)$	$-\cos x$	$\sin x$	$x \log x - x$	$3\ e^{3x}$	$x^7 + 2x^3$

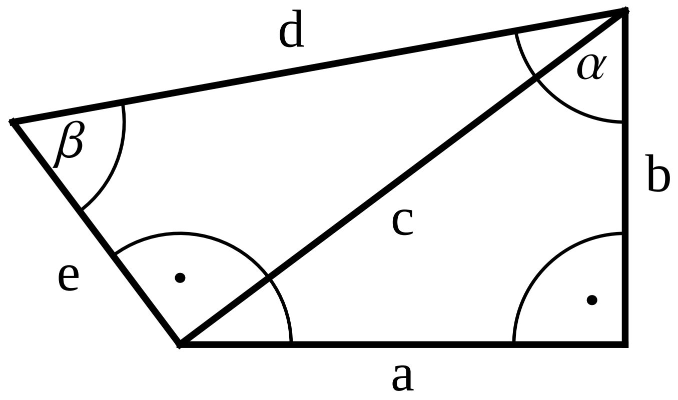
(b) Compute the area of the region which is enclosed by the graphs of $f(x) = -6x^2 + 12x + 3$ and $g(x) = 4x^3 - 12x^2 + 8x + 3$ between $x = 0$ and $x = 2$ (see below).



Answer: The shaded area is the difference of the area enclosed by the graph of $f(x)$ and the x-axis and the one enclosed by the graph of $g(x)$ and the x-axis. Thus,

$$A = \int_0^2 dx\ (f(x) - g(x)) = \left[\left(-2x^3 + 6x^2 + 3x\right) - \left(x^4 - 4x^3 + 4x^2 + 3x\right)\right]_{x=0}^{x=2} = 8$$

3. Trigonometric Functions



Referring to the figure above fill in the blanks:

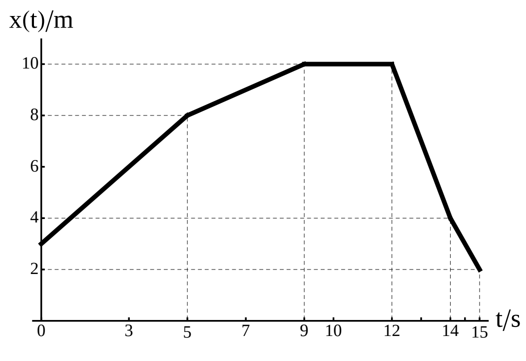
$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\sin\beta$	$\cos\beta$	$\tan\beta$
a/c	b/c	a/b	c/d	e/d	c/e

And again, fill in the blanks:

α	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π
$\sin\alpha$	0	$\frac{1}{2}\sqrt{2}$	1	0
$\cos\alpha$	1	$\frac{1}{2}\sqrt{2}$	0	-1
$\tan\alpha$	0	1	–	0

4. Kinetics in 1 Dimension

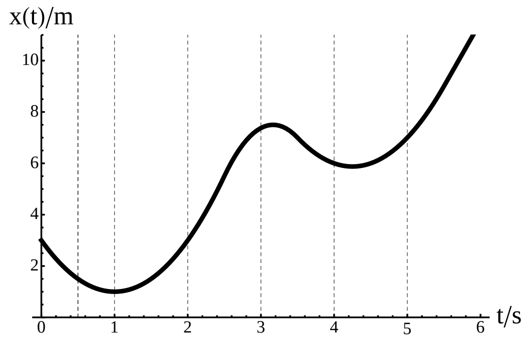
(a) We consider a particle which moves along a line. The following diagram shows its position $x(t)$ at time t.



Give the velocities at the respective times.

t/s	3	7	10	13	14.5
$v(t)$/(m/s)	1	1/2	0	-3	-1/2

(b) Consider now the motion depicted by the following diagram.



What is the sign (positive $+$, negative $-$, or zero 0) of velocity $v(t)$ and acceleration $a(t)$ at the respective times t? Fill in the blanks!

$t/$ s	0.5	1	2	3	4	5
sign $v(t)$	-	0	+	+	-	+
sign $a(t)$	+	+	+	-	+	+

Remark: The plot was slightly modified such that it is now easier to see that $a(5\,\mathrm{s}) > 0$.

5. Kinetics in 2 Dimensions

We consider the fate of an ant which, at time $t = 0s$, is located in the center of a room, which in cartesian coordinates corresponds to $(0\,\mathrm{m}, 0\,\mathrm{m})$. During the first second, $t \in (0\,\mathrm{s}, 1\,\mathrm{s}]$, the ant moves with velocity $\vec{v}_1$. In the following two, $t \in (1\,\mathrm{s}, 3\,\mathrm{s}]$ it moves with velocity $\vec{v}_2$, and in the time interval $t \in (3\,\mathrm{s}, 5\,\mathrm{s}]$ with $\vec{v}_3$. Then, taking a break, it rests for 2 s at the same point. Afterwards, starting with velocity 0 m/s, it moves with constant acceleration $\vec{a}$, stopping after 1 s. The aforementioned velocities and acceleration are

$$\vec{v}_1 = \begin{pmatrix} 1\,\mathrm{m/s} \\ 0\,\mathrm{m/s} \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 0\,\mathrm{m/s} \\ 1\,\mathrm{m/s} \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 1\,\mathrm{m/s} \\ -1\,\mathrm{m/s} \end{pmatrix}, \quad \vec{a} = \begin{pmatrix} 0\,\mathrm{m/s^2} \\ -2\,\mathrm{m/s^2} \end{pmatrix}.$$

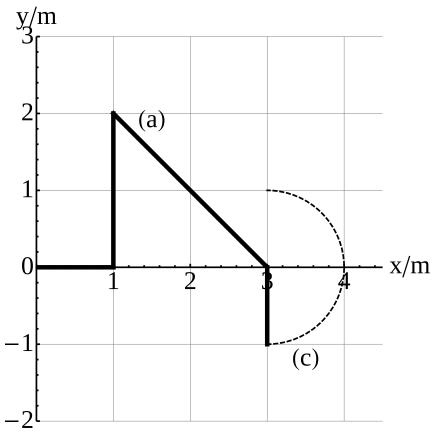
(a) Sketch the path of the ant in the x-y-plane.

(b) Plot the coordinate $y(t)$ as well as the y-component of the velocity vector $\vec{v}(t)$ as a function of time.

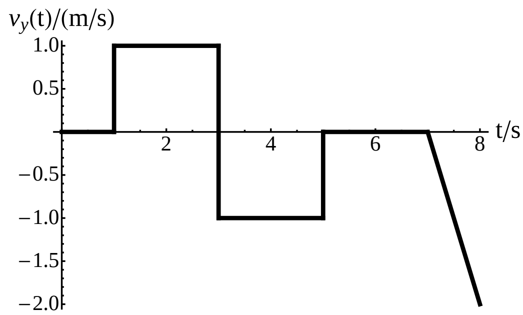
(c) Once the ant reaches the point $(3\,\mathrm{m}, -1\,\mathrm{m})$ at time $t = 8\,\mathrm{s}$, it starts moving counter-clockwise along a circle with constant velocity $|\vec{v}(t)| = v_0$ (the direction, of course, changes!). The circle's center is $(3\,\mathrm{m}, 0\,\mathrm{m})$, its radius is $r = 1\,\mathrm{m}$. After 2 s the ant reaches the point $(3\,\mathrm{m}, 1\,\mathrm{m})$.

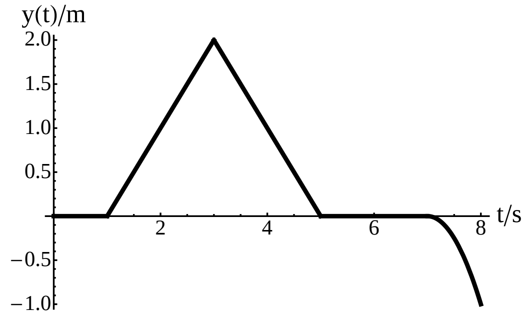
Sketch the new path. What is the ant's velocity v_0? Plot the components of the velocity vector $\vec{v}(t)$ for $t \in [8\,\mathrm{s}, 10\,\mathrm{s}]$.

Answer:



(b) Plots:

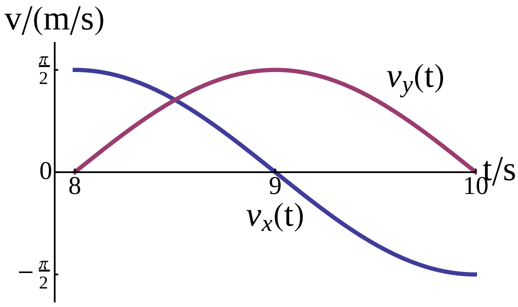




(c) Beginning at $t = 8\,\mathrm{s}$ the ant performs a circular motion. Apparently (see figure above), it travels half of the circumference $U = 2\pi r$ of the circle, which corresponds to a distance of $l = \pi r = \pi \cdot 1\,\mathrm{m}$. Since it needs $\Delta t = 2\,\mathrm{s}$, its velocity is $v_0 = l/\Delta t = \frac{\pi}{2}\,\mathrm{m/s}$.
The same result can be obtained in a more general way. We know that a circular motion can be parametrized as follows: $\vec{r}(t) = \begin{pmatrix} x_0 + r\cos(\phi(t)) \\ y_0 + r\sin(\phi(t)) \end{pmatrix}$. Here $\vec{r}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ is the center of the orbit and r is its radius. Since the absolute value of the (linear) velocity $|\vec{v}(t)|$ is constant, so is the angular velocity $\dot{\phi}(t) = \omega = const.$ That means $\phi(t) = \omega t + \phi_0$.
In the case of our ant, we have $x_0 = 3\,\mathrm{m}$, $y_0 = -1\,\mathrm{m}$, $r = 1\,\mathrm{m}$. Furthermore we know that $\phi(8\,\mathrm{s}) = \omega \cdot 8\,\mathrm{s} + \phi_0 = -\pi/2$ and $\phi(10\,\mathrm{s}) = \omega \cdot 10\,\mathrm{s} + \phi_0 = \pi/2$. Solving these two equations gives $\phi_0 = -\frac{9}{2}\pi$, $\omega = \frac{\pi}{2}\,\mathrm{s}^{-1}$. Hence, the velocity is

$$\vec{v}(t) = \dot{\vec{r}}(t) = \begin{pmatrix} -r\dot{\phi}(t)\ \sin\phi(t) \\ r\dot{\phi}(t)\ \cos\phi(t) \end{pmatrix} = \begin{pmatrix} -\frac{\pi}{2}\frac{\mathrm{m}}{\mathrm{s}}\sin\phi(t) \\ \frac{\pi}{2}\frac{\mathrm{m}}{\mathrm{s}}\cos\phi(t) \end{pmatrix}$$

with $\phi(t) = \frac{\pi}{2}\ t/\mathrm{s} - \frac{9}{2}\pi$. Due to the identity $\cos^2 x + \sin^2 x = 1$, it is $v_0 = |\vec{v}(t)| = |r\omega| = \frac{\pi}{2}\mathrm{m/s}$, the same result as we obtained before. As a byproduct we additionally know the complete time dependence of $\vec{v}(t)$.



v/(m/s)
π/2
0
−π/2
8
9
10
t/s
v_y(t)
v_x(t)