Karlsruhe Institute of Technology - International Department GmbH

Pre-Semester 2010 - Physics Course - Extra Tutorial

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Solution 11
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1. Lorentz Force

In a constant magnetic field $\vec{B}$ which points upwards (in y-direction), a point with charge $q > 0$ moves with velocity $\vec{v}$ to the right (along the x-axis) .

(a) In which direction does the Lorentz force $\vec{F}_L$ point?

(b) What is its magnitude F_L?

(c) Suppose you want to compensate the force exerted by $\vec{B}$ via an electric field $\vec{E}$. In which direction does it have to point and what magnitude E should it have?

(d) Calculate F_L and E for $q = 1.6 \cdot 10^{-19}\,\text{C}$, $v = 10^5\,\text{m/s}$, and $B = 100\,\text{mT}$.

Solution:

(a) According to the *right-hand-rule* $\vec{F}_L = q\vec{v} \times \vec{B}$ points out of the paper plane (in z-direction).

(b) Since $\vec{v} \perp \vec{B}$: $F_L = qvB$.

(c) The forces compensate each other if

$$q\vec{E} = -q\vec{v} \times \vec{B} \quad \Leftrightarrow \quad \vec{E} = -\vec{v} \times \vec{B}.$$

Thus $\vec{E}$ points into the paper plane (in *negative* z-direction) and $E = vB$

(d) $F_L = 1.6 \cdot 10^{-16}\,\text{N}$, $E = 10^4\,\text{V/m}$.

2. Two Coils

Consider two coils - a small one inside a larger one - having the same axis. The larger one has length ℓ_1 and consists of N_1 turns. An alternating current $I(t) = I_0 \cos(\omega t)$ is flowing through it.

The smaller coil has N_2 turns, its cross section is A_2. What voltage $U_{\text{ind}}(t)$ is induced between its ends? *Remark:* Assume that no current flows through the small coil and neglect all effects that it may have on the magnetic field produced by the large one.

Solution: We do the calculation in 3 steps:

(i) <u>Magnetic field $B(t)$ produced by large coil:</u>

$$B(t) = \mu_0 \frac{N_1}{\ell_1} I(t) = \mu_0 \frac{N_1}{\ell_1} I_0 \cos(\omega t)$$

(ii) Voltage $U_{\text{ind},0}(t)$ induced in *each of the turns* of small coil:
Flux through each turn (each of them is a loop with area A_2): $\Phi_0(t) = A_2 B(t)$. Faraday's law of induction gives

$$U_{\text{ind},0}(t) = -\dot{\Phi}_0(t) = -A_2 \dot{B}(t) = \mu_0 \frac{N_1 A_2}{\ell_1} I_0 \omega \sin(\omega t).$$

(iii) Voltage $U_{\text{ind}}(t)$ induced in complete small coil:
The coil is a "series connection" of N_2 loops. Their induced voltages $U_{\text{ind},0}$ thus add up:

$$U_{\text{ind}}(t) = N_2 U_{\text{ind},0}(t) = \mu_0 \frac{N_1 N_2 A_2}{\ell_1} I_0 \omega \sin(\omega t).$$

3. Loop in Magnetic Field

Consider a wire in shape of a rectangle which may rotate around one of its axes. The edges parallel to the rotation axis each have length $\ell = 10\,\text{cm}$, the perpendicular edges each have length $d = 8\,\text{cm}$. The wire carries a current $I = 100\,\text{mA}$. A magnetic field $B = 1\,\text{mT}$ perpendicular to the rectangle's rotation axis is present and exerts a moment of torque of $M = 4 \cdot 10^{-7}\,\text{Nm}$ on the wire. What is the angle α that the rectangle makes with the magnetic field?

Solution: We do the calculation in 3 steps:

(i) Force F_l on each of parallel edges:

$$F_l = I\ell B = 10^{-5}\,\text{N}.$$

Remark: The forces F_d on the perpendicular edges are parallel to the rotation axis and do not exert any moment of torque.

(ii) Moment of torque M for given angle α between rectangular loop and $\vec{B}$:
On each of the two parallel edges a force $\vec{F}_1$ or $\vec{F}_2$ acts (with $|\vec{F}_1| = F_l = |\vec{F}_2|$). If we denote the arms which connect the rotation axis to the edges by $\vec{r}_1$ and $\vec{r}_2$ (with $|\vec{r}_1| = \frac{d}{2} = |\vec{r}_2|$), the total moment of torque is the sum $\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$. Apparently, $\vec{r}_2 = -\vec{r}_1$, $\vec{F}_2 = -\vec{F}_1 \Rightarrow \vec{r}_2 \times \vec{F}_2 = \vec{r}_1 \times \vec{F}_1 \Rightarrow \vec{M} = 2\vec{r}_1 \times \vec{F}_1$. If we denote the angle between $\vec{r}_1$ and $\vec{F}_1$ by β, we have

$$M = 2 \cdot \frac{d}{2} F_l \sin\beta = dI\ell B \cos\alpha$$

where we have used that the angle α between rectangular loop and $\vec{B}$ is $\alpha = 90° - \beta$.

(iii) Angle α for given M:

$$\cos\alpha = \frac{M}{dI\ell B} \qquad \Rightarrow \qquad \alpha = \arccos\frac{M}{dI\ell B} = 60°$$

4. Rotating Bar-Generator

A conducting bar of length r rotates with angular frequency ω around a pivot P at one end of the bar. The other end of the bar is in slipping contact with a stationary wire in the shape of a circle. Point P and the wire are connected via a resistor R. Thus the bar, the resistor and the wire form a closed conducting loop. The resistance of the bar and the circular wire are negligibly small. There is a uniform magnetic field B everywhere, it is perpendicular to the plane of the paper. What is the induced current in the loop?

Solution: We do the calculation in 4 steps:

(i) Area A of loop for given angle α between bar and resistor:

$$A = \frac{\alpha}{2\pi}\pi r^2 = \frac{\alpha}{2}r^2.$$

(ii) Magnetic flux Φ through loop:

$$\Phi = BA = \frac{\alpha}{2}Br^2.$$

(iii) Induced voltage Φ is time-dependent because α is time-dependent with $\dot{\alpha}(t) = \omega$! Faraday's law of induction gives:

$$U_{\text{ind}}(t) = -\dot{\Phi}(t) = -\frac{\dot{\alpha}}{2}Br^2 = -\frac{\omega}{2}Br^2.$$

(iv) Current I: Ohm's law states

$$I = U_{\text{ind}}/R = -\frac{\omega Br^2}{2R}.$$

Remark: Don't bother about the question how exactly the current flows *through the circular wire*. Assuming that the magnetic field points out of the plane and the bar is rotated counterclockwise, Lenz's law definitely states that the current I flows away from P *in the bar*, and towards P *through the resistor*.