Karlsruhe Institute of Technology - International Department GmbH

Pre-Semester 2010 - Physics Course - Extra Tutorial

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Solution 12
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1. Travelling Point

Consider the following journey of a point mass with charge $q = 1.6 \cdot 10^{-19}\,\text{C}$:

(1) Initially at rest, it is accelerated in passing a parallel-plate capacitor with voltage U_1.

(2) Afterwards, it flies through a velocity selector with crossed electric and magnetic fields ($E_2 = 5.93 \cdot 10^4\,\text{V/m}$, $B_2 = 100\,\text{mT}$) without being deflected.

(3) Finally, it enters a region with constant magnetic field $B_3 = 10^{-4}\text{T}$. Since its velocity is not perpendicular to $\vec{B}_3$ it performs a helical motion ("winding around the magnetic field lines") with angular frequency $\omega = 17.56 \cdot 10^6\,\text{s}^{-1}$.

Assuming that the point's velocity does not change *between* the 3 regions, calculate U_1!

Solution:

(1) In the accelerating capacitor we have *energy conservation*:

$$\frac{1}{2}mv^2 = qU_1 \quad \Rightarrow \quad U_1 = \frac{mv^2}{2q}.$$

Thus, we could calculate U_1, if we knew m and v. It may help to have a look at the other regions.

(2) Since the point mass is not deflected in the velocity selector, the forces $F_e = qE_2$ and $F_m = qvB_2$ due to the electric and magnetic field compensate each other. Hence,

$$v = \frac{E_2}{B_2} = 5.93 \cdot 10^5\text{m/s}.$$

(3) By decomposing the velocity $\vec{v} = \vec{v}_\parallel + \vec{v}_\perp$ ($\vec{v}_\parallel \parallel \vec{B}_3$, $\vec{v}_\perp \perp \vec{B}_3$) we see that the charged mass in the inclined magnetic field B_3 performs an helical motion which is a superposition of (a) a uniform motion *parallel* to $\vec{B}_3$ and (b) a circular motion *perpendicular* to $\vec{B}_3$. The radius of the latter is $r_\perp = \frac{mv_\perp}{qB_3}$ and the angular velocity is $\omega = v_\perp / r_\perp = \frac{qB_3}{m}$. Hence, the mass is

$$m = \frac{qB_3}{\omega} = 9.11 \cdot 10^{-31}\,\text{kg}.$$

Putting all together, we get $U_1 = 1\,\text{V}$.

2. Another Generator

Consider the generator drawn on the blackboard: In a magnetic field B, a conducting "open rectangle" with edge lengthes l (parallel to rotation axis) and d (perpendicular to rotation axis) is rotated with constant angular velocity ω. The two perpendicular edges are connected via a resistor R. Initially, the rectangle is perpendicular to the magnetic field, say, such that the enclosed flux is minimal. Neglect self-inductance effects (that means neglect the field which the current through the rectangle produces).

(a) What is the induced current $I(t)$ as a function of time t?

(b) What power $P_{\text{el}}(t)$ is dissipated in the resistor?

(c) What moment of torque $M(t)$ is needed to keep the rectangle rotating with constant angular velocity?

(d) With what mechanical power $P_{\text{mech}}(t)$ does the generator, hence, have to be supplied?

Solution:

(a) If α is the angle between the turning rectangle's normal vector and $\vec{B}$, such that, initially, $\alpha(t=0)=0$, then $\alpha(t)=\omega t$. First, we look for the total *flux* $\Phi = AB_\perp$ or $\Phi = A_\perp B$ (think about it! they're the same) through the circuit. That means, we need its area $A_\perp$ perpendicular to $\vec{B}$.
In fact $A_\perp$ will depend on α where it is required that $A_\perp(\alpha)$ is minimal for $t=0$ or $\alpha=0$. Let us define $A_0 = A_\perp(\alpha=\pi)$, then: $A_\perp(\alpha) = A_0 - ld\cos(\alpha)$. ($ld$ is the area of the turning rectangle). The flux, thus is

$$\Phi(t) = BA_\perp(\alpha(t)) = B(A_0 - ld\cos(\omega t)).$$

According to Faraday's law of induction and Ohm's law we get:

$$U_{\text{ind}}(t) = -\dot{\Phi}(t) = -Bld\omega\sin(\omega t),$$
$$I(t) = U_{\text{ind}}(t)/R = -\frac{Bld}{R}\omega\sin(\omega t).$$

(b) The dissipated power is

$$P_{\text{el}}(t) = U_{\text{ind}}(t)^2/R = \frac{B^2l^2d^2\omega^2}{R}\sin(\omega t)^2.$$

(c) To keep the rectangle rotating (against the Lorentz force which tries to slow it down) one needs the opposing force $F_{\text{mech}} = F_L = IlB$ whose component perpendicular to the arm is $F_{\text{mech},\perp} = F_{\text{mech}}\sin\alpha$. The moment of torque, thus, is

$$M = F_{\text{mech},\perp}d = IlBd\sin\alpha \quad \Rightarrow \quad M(t) = (-)\frac{B^2l^2d^2}{R}\omega\sin(\omega t).$$

Dont't worry about the sign! One could have used clever arguments to get rid of it. But to assure you how unimportant it is, I will just ignore it.

(d) The mechanical power is

$$P_{\text{mech}}(t) = M(t)\omega = \frac{B^2l^2d^2}{R}\omega^2\sin(\omega t)^2.$$

3. **Crossbar on Rails**

Two parallel conducting rails with distance d make an angle α with the horizontal and are connected via a resistor R. A conducting crossbar with mass m lies perpendicular on both rails and may slide along them without friction. Because of gravity (acting downwards, in vertical direction) the crossbar moves downwards. But because there is a constant vertical magnetic field B which exerts the Lorentz force, the crossbar moves with constant velocity v. Neglecting self-inductance, determine v!

Solution:

(i) Lorentz force F_L necessary to compensate gravity (since the crossbar should move with constant velocity v):

$$F_L = mg \tan\alpha.$$

(ii) Current I
Lorentz force $F_L = IdB$

$$\Rightarrow I = \frac{F_L}{dB} = \frac{mg}{dB} \tan\alpha.$$

(iii) Induced voltage U_{ind}
Ohm's law: $U_{\text{ind}} = RI = R\frac{mg}{dB} \tan\alpha$.

(iv) Velocity v necessary to induce U_{ind}
Flux $\Phi = Bsd \cos\alpha$ with $\dot{s} = -v$
Faraday's law of induction: $U_{\text{ind}} = -\dot{\Phi} = Bdv \cos\alpha$

$$\Rightarrow v = \frac{U_{\text{ind}}}{dB \cos\alpha} = R\frac{mg}{d^2B^2}\frac{\tan\alpha}{\cos\alpha}.$$