Karlsruhe Institute of Technology - International Department GmbH

Pre-Semester 2010 - Physics Course - Extra Tutorial

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Solution 2
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1. Newton's axioms

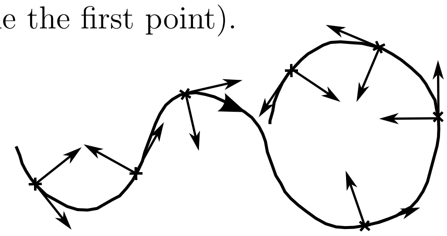
(a) State Newtons's first axiom (the principle of inertia)!

Hence, is there a force acting on a body which performs a circular motion?

Answer: Any object moves with constant velocity $\vec{v}$, in absolute value *and* direction, (possibly $\vec{v} = \vec{0}$ for a resting object) *unless* acted on by a net external force. Since during a circular motion the direction of $\vec{v}$ changes with time, there has to be a force acting on the object.

(b) State Newtons's second axiom (the principle of action)!

Consider a particle which moves along the path given below, with velocity $|\vec{v}(t)| = const$. Sketch the direction of the velocities and forces which act in the marked points (see as an example the first point).



Answer: Any object with mass m on which a net external force $\vec{F}$ acts undergoes an acceleration $\vec{a}$ which is proportional to $\vec{F}$ and inversely proportional to m: $\vec{F} = m\vec{a}$.

(c) State Newton's third axiom (*actio=reactio*)!

Imagine an apple (with mass $m = 200\,\text{g}$) falling from a tree. What is the gravitational force with which it attracts the earth (mass $M = 6 \cdot 10^{24}\,\text{kg}$)? What is, hence, the acceleration with which the earth falls towards the apple?

Answer: If object A exerts a force $\vec{F}_{AB}$ on object B, an equal but opposite force $\vec{F}_{BA} = -\vec{F}_{AB}$ is exerted on object A by object B.

The force with which the earth attracts the apple is the weight of the latter, $F = mg = 5\,\text{N}$, pointing from the apple to the earth's center. The apple attracts the earth with an equal force $5\,\text{N}$, pointing in the opposite direction. But, since the earth is much heavier than the apple, it undergoes a much smaller acceleration $a = F/M = \frac{m}{M}g \approx 3 \cdot 10^{-26}\,\text{m/s}^2$.

2. Miscellaneous

(a) A bullet of mass $1.8 \cdot 10^{-3}\,\text{kg}$ moving at $500\,\text{m/s}$ impacts a large fixed block of wood and travels $6\,\text{cm}$ before coming to rest. Assuming that the acceleration of the bullet is constant, find the force exerted by the wood on the bullet.

Answer: We describe the system using an one dimensional coordinate system such that the bullet is coming from the left with velocity $v = 500\,\mathrm{m/s}$ and hits the wood block at position $x_i = 0\,\mathrm{cm}$ and at time $t_i = 0\,\mathrm{cm}$. Then it stops at $x_s = 6\,\mathrm{cm}$, at the unknown time t_s. Since we assume a constant acceleration $a = F/m < 0$, slowing down the bullet, for times $t_i \leq t \leq t_s$ we use the general ansatz

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2.$$

The velocity then is $v(t) = \dot{x}(t) = v_0 + at$.
The relations $x(t_i) = x_i$ and $v(t_i) = v$ fix $x_0 = x_i$ and $v_0 = v$ such that $x(t) = \frac{1}{2}at^2 + vt$ and $v(t) = v_0 + at$. Further, we know that the bullet stops at the time t_s, that means $v(t_s) = 0$, after it has travelled a certain distance, $x(t_s) = x_s$. Combining the two equations gives $t_s = -v/a$ and $x_s = x(t_s) = -\frac{1}{2}v^2/a$. Hence, the acceleration was $a = -\frac{1}{2}v^2/x_s$ and the force $F = ma = -\frac{1}{2}mv^2/x_s = -3125\,\mathrm{N}$.

A more clever way to obtain the same result, which relies on energy considerations, is the following: The initial kinetic energy of the bullet is $E_0 = \frac{1}{2}mv^2$, after the impact the wood block exerts a constant force F over a distance x_s, thus doing the work $W = Fx_s$ which adds to the initial energy of the bullet. Since in the end the bullet is at rest, it has no (kinetic) energy. Thus, $0 = E_0 + W \Rightarrow F = -\frac{1}{2}mv^2/x_s$.

(b) To drag a 75-kg log along the ground at constant velocity, you have to pull on it with a horizontal force of 250 N.

(i) What is the resistive force exerted by the ground?

Answer: Since the log moves with constant velocity, the net force has to be zero, $F_{\mathrm{net}} = F_{\mathrm{pull}} + F_{\mathrm{res}} = 0$. $F_{\mathrm{pull}} = 250\,\mathrm{N}$ is the force with which you pull in forward direction, F_{res} is the resistive force in opposite direction. $\Rightarrow F_{\mathrm{res}} = -250\,\mathrm{N}$.

(ii) What horizontal force must you exert if you want to give the log an acceleration of $2\,\mathrm{m/s^2}$?

Answer: According to Newton's second axiom the net force now has to be $F'_{\mathrm{net}} = ma = 150\,\mathrm{N}$. Assuming F_{res} hasn't changed, the log has to be pulled on with a force of $F'_{\mathrm{pull}} = F'_{\mathrm{net}} - F_{\mathrm{res}} = 400\,\mathrm{N}$.

3. Hook's Law

A vertical spring of force constant 600 N/m has one end attached to the ceiling and the other to a 12-kg block resting on a horizontal surface so that the spring exerts an upward force on the block. The spring stretches by 10 cm.

(i) What force does the spring exert on the block?

Answer: According to Hook's law, the force of the spring (force constant D, stretched by Δx) is $F_s = +|D\Delta x| = 60\,\mathrm{N}$. We have chosen upward forces to be positive and downward forces to be negative.

(ii) What is the force that the surface exerts on the block?

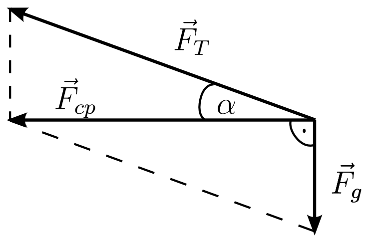
Answer: There are three forces which act on the block: one is F_s, the second is its weight, which points downward, $F_g = -mg = -120\,\mathrm{N}$. The third, F_n, is exerted upward by the surface preventing the block from falling to the ground. Since the net force vanishes (the block rests), $0 = F_s + F_g + F_n$, it is $F_n = -F_s - F_g = 60\,\mathrm{N}$.

4. **Centripetal Force**

A 0.20-kg-stone attached to a 0.8-m string is rotated in the horizontal plane. The string makes an angle of 20° with the horizontal. Determine the speed of the stone and the tension of the string.

Answer: In the described situation (draw a picture!) the stone moves along a horizontal circle with radius $r = l\cos(\alpha)$ (where $l = 0.8\,\mathrm{m}$ is the length of the string and $\alpha = 20°$ is its angle with the horizontal). In order to keep the stone on this circular orbit the centripetal force $|\vec{F}_{cp}| = mv^2r$ has to act on the stone (where v is its velocity and $m = 0.2\,\mathrm{kg}$ is its mass) which points towards the circle's center. Where is this force coming from?

In fact, there are to forces which act on the stone. The first one, $\vec{F}_T$, is exerted by the rope. It points towards the suspension (the point where the rope is fixed at the ceiling). The second one is the stone's weight, $|\vec{F}_g| = mg = 2\,\mathrm{N}$, pointing downward. Their sum is $\vec{F}_T + \vec{F}_g = \vec{F}_{cp}$:



Hence, $|\vec{F}_g| = |\vec{F}_T|\sin(\alpha)$ and $|\vec{F}_{cp}| = |\vec{F}_T|\cos(\alpha)$. This yields $|\vec{F}_T| = mg/\sin(\alpha) = 5.8\,\mathrm{N}$ and

$$\frac{\cos(\alpha)}{\sin(\alpha)} = \frac{|\vec{F}_{cp}|}{|\vec{F}_g|} = \frac{v^2}{gr} \Rightarrow v = \sqrt{gr\frac{\cos(\alpha)}{\sin(\alpha)}} = \sqrt{gl\frac{\cos(\alpha)^2}{\sin(\alpha)}} = 4.5\,\mathrm{m/s}.$$