Karlsruhe Institute of Technology - International Department GmbH

Pre-Semester 2010 - Physics Course - Extra Tutorial

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Solution 5
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1. Moment of Inertia of a Hammer

We consider a hammer which consists of two perpendicular rods with masses $m_1 = 0.5\,\mathrm{kg}$ and $m_2 = 1.5\,\mathrm{kg}$, and lengths $l_1 = 0.3\,\mathrm{m}$ and $l_2 = 0.2\,\mathrm{m}$ (neglect their lateral extent). One of the ends of the first rod is attached to the center of the second rod. Calculate the moment of inertia Θ of the hammer with respect to the axis which goes through the first rod's other end and which is perpendicular to both rods.

Note: The moment of inertia of a rod of length l_r (and negligible lateral extent) and mass m_r with respect to an axis through the center of mass and perpendicular to the rod is

$$\Theta_{\mathrm{cm}} = \frac{1}{12} m_r l_r^2.$$

Solution: The moments of inertia of the two rods with respect to the respective centers of mass is

$$\Theta_{\mathrm{cm},1} = \frac{1}{12} m_1 l_1^2, \quad \Theta_{\mathrm{cm},2} = \frac{1}{12} m_2 l_2^2.$$

The parallel-axis theorem gives for the moments of inertia with respect to the rotation axis defined above:

$$\Theta_1 = \Theta_{\mathrm{cm},1} + m_1 \left(\frac{l_1}{2}\right)^2 = \frac{1}{3} m_1 l_1^2,$$

$$\Theta_2 = \Theta_{\mathrm{cm},2} + m_2 l_1^2 = \left(\frac{1}{12} l_2^2 + l_1^2\right) m_2.$$

Their sum is the moment of inertia of the hammer,

$$\Theta = \Theta_1 + \Theta_2 = \frac{1}{3} m_1 l_1^2 + \left(\frac{1}{12} l_2^2 + l_1^2\right) m_2 = 0.155\,\mathrm{kg\,m^2}$$

2. Parallel-Axis Theorem

Consider a rigid body of mass $m = 1\,\mathrm{kg}$, and two possible axes which are parallel to each other. Their distances to the body's center of mass are $s_1 = 1\,\mathrm{m}$ and $s_2 = 2\,\mathrm{m}$. If the body's moment of inertia with respect to the first axis is $\Theta_1 = 5\,\mathrm{kg\,m^2}$, what is its moment of inertia with respect to the second axis?

Solution: If we denote by Θ_{cm} the moment of inertia with respect to an axis through the center of mass and parallel to the given two, the parallel-axis theorem gives

$$\Theta_1 = \Theta_{\mathrm{cm}} + m s_1^2 \qquad \Rightarrow \Theta_{\mathrm{cm}} = \Theta_1 - m s_1^2,$$

$$\Theta_2 = \Theta_{\mathrm{cm}} + m s_2^2 \qquad \Rightarrow \Theta_2 = \Theta_1 - m s_1^2 + m s_2^2 = 8\,\mathrm{kg\,m^2}.$$

3. Physical Pendulum

Consider a thin rod of length $l = 2\,\mathrm{m}$ (neglect its lateral extent) and (uniformly distributed) mass $m = 3\,\mathrm{kg}$. Two point masses are attached to it: the first one, $m_1 = 4\,\mathrm{kg}$, in the middle, and the second one, $m_2 = 1\,\mathrm{kg}$ at one of its ends. At its other end the rod is hinged such that it may rotate around a perpendicular axis.

(a) With respect to the described axis, what is the moment of inertia Θ of the given pendulum?

(b) What is the moment of torque M acting on the pendulum due to gravity, if it makes an angle α with the vertical?

(c) Give the equation of motion (for α) and approximate it for small α.

Solution:

(a) Using the parallel-axis theorem one obtains for the rod (see Ex. 1): $\Theta_{\mathrm{rod}} = 1/12\, ml^2 + m(l/2)^2 = 1/3\; ml^2$. Thus,

$$\Theta = \Theta_{\mathrm{rod}} + m_1 \left(\frac{l}{2}\right)^2 + m_2 l^2 = \frac{1}{3} m l^2 + m_1 \left(\frac{l}{2}\right)^2 + m_2 l^2 = 12\,\mathrm{kg\,m}^2.$$

(b) We choose the signs as follows: If the pendulum is deflected to the right (i. e. counter-clockwise), the angle α is positive. If its deflected to the left, α is negative. If $\vec{L}$, $\vec{M}$, $\vec{\omega}$ point out of the blackboard, then L, M, ω are positive. If $\vec{L}$, $\vec{M}$, $\vec{\omega}$ point into the blackboard, then L, M, ω are negative. Then, $\omega = \dot{\alpha}$.

The moment of torque thus is (gravity effectively acts on the centers of mass)

$$M = -(ml/2 + m_1 l/2 + m_2 l) g \sin\alpha = -90\,\mathrm{kg\,m}^2/\mathrm{s}^2\;\; \sin\alpha.$$

(c) Newton's second law for angular quantities reads $\dot{L} = M$. With $L = \Theta\omega = \Theta\dot{\alpha}$, the equation of motion reads

$$12\,\mathrm{kg\,m}^2\;\ddot{\alpha} + 90\,\mathrm{kg\,m}^2/\mathrm{s}^2\;\; \sin\alpha = 0.$$

For small angles α, it is $\sin\alpha \approx \alpha$, and the equation simplifies to

$$12\,\mathrm{kg\,m}^2\;\ddot{\alpha} + 90\,\mathrm{kg\,m}^2/\mathrm{s}^2\;\; \alpha = 0.$$