Karlsruhe Institute of Technology - International Department GmbH

Pre-Semester 2010 - Physics Course - Extra Tutorial

Stéphane Ngo Dinh
stephane.ngodinh@kit.edu

Solution 8
08.09.2010

---

1. Electrostatics - Things to *learn by heart*

(a) **Coulomb's law**: What force F_{12} does a point with charge q_1 exert on a point with charge q_2 which is located in a distance r_{12}? In which direction does it point?

(b) An **electric field** $\vec{E}$ is *defined* (it's best not to ask for a deeper meaning) by the force $\vec{F}$ which it exerts on a charge q. How are the 3 quantities related? For given $\vec{E}$ and q, in which direction does the force $\vec{F}$ point?

(c) Imagine two large **parallel plates** with distance d, each of them having an area A. One of them is charged with (constant) *charge density* $\boxed{\sigma = Q/A}$, the other one is charged oppositely with $-\sigma$. What is the electric field between the two plates? In which direction does it point? What is the **voltage** U between the two plates?

Solution:

(a) Coulomb's law (p. 43, Eq. (5.7)):

$$|F_{12}| = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{(r_{12})^2}.$$

If q_1 and q_2 have the same sign (both positive or both negative), they *repel* each other. That means, the force acting on q_2 points *away from* q_1. If they have the opposite sign, they *attract* each other, and the force acting on q_2 points *towards* q_2.

(b) Electric field (p. 43, eq. (5.5)):

$$\boxed{\vec{E} = \vec{F}/q} \qquad \Leftrightarrow \qquad \boxed{\vec{F} = q\vec{E}}$$

$\vec{F}$ is parallel to $\vec{E}$:

- If q is *positive*, then $\vec{F}$ points in the same direction.
- I q is *negative*, then $\vec{F}$ points in the opposite direction.

(c) Two parallel plates (p. 44, eq. (5.8)): $\boxed{E = \sigma/\epsilon_0}$. $\vec{E}$ points *from positive to negative plate*.
Voltage (p. 45, eq. (5.13)): $\boxed{U = Ed} \Leftrightarrow \boxed{E = U/d}$.

$$\Rightarrow U = Q\frac{d}{\epsilon_0 A}$$

2. Potential energy and electric voltage

(a) A point with mass m moves under the influence of gravity g. Initially (at $t = 0$) it is at rest and located at an height h above the ground.

(i) What force F_g acts on the point? In which direction does it point?

(ii) What is the potential energy of the point if it is at an height z above the ground?

(iii) What is its velocity v_g when it hits the ground?

(b) A point with mass m and charge $q < 0$ moves through an homogeneous electric field $E > 0$ which points upwards. Gravity is neglected. Initially (at $t = 0$) the point is at rest and located at an height $z(t = 0) = h$ above the ground.

(i) What force F acts on the point? In which direction does it point?

(ii) What is the point's position $z(t)$ at time t?

(iii) What is the point's velocity $v(t)$ at time t?

(iv) At what time t_g does the point hit the ground?

(v) What is its velocity v_g when it hits the ground?

(vi) Let's assume the **potential energy** of the point is $\boxed{E_{\text{pot}} = -qEz}$, if it is at an height z above the ground. Assume further that the total energy $E_{\text{pot}} + E_{\text{kin}}$ is conserved.
Again, calculate the velocity v_g with which the point hits the ground!

(vii) The **electric voltage** *between two points* z_1 and z_2 is $\boxed{U_{12} = (E_{\text{pot}}(z_1) - E_{\text{pot}}(z_2))/q}$.
Calculate the voltage U_h between $z = h$ and the ground. Does it depend on q?

Solution:

(a) (i) Weight $F_g = -mg$, points downwards. To be consistent with part (b) I put in an additional minus sign, which just means, that the force points downwards.

(ii) $E_{\text{pot}}(z) = mgz$

(iii) Energy conservation $\Rightarrow mgh = \frac{1}{2}mv_g^2 \Rightarrow v_g = \sqrt{2gh}$.

(b) (i) $F = qE = -|q|E < 0$. q is negative $\Rightarrow \vec{F}$ opposite to $\vec{E} \Rightarrow \vec{F}$ points downwards (consistent with $F < 0$).

(ii) Motion with *constant acceleration*

$$\ddot{z}(t) = a = F/m = -|q|E/m \ (< 0)$$

and initial conditions

$$z(t = 0) = h, \quad \dot{z}(t = 0) = 0.$$

The solution is

$$z(t) = -\frac{1}{2}\frac{|q|E}{m}t^2 + h.$$

It won't hurt if you check it for yourself.

(iii) $v(t) = \dot{z}(t) = -\frac{|q|E}{m}t$

(iv) The time of impact t_g is defined by $z(t_g) = 0$. That means

$$-\frac{1}{2}\frac{|q|E}{m}t_g^2 + h = 0 \qquad \Leftrightarrow \qquad t_g = \sqrt{-\frac{2hm}{qE}} = \sqrt{\frac{2hm}{|q|E}}.$$

(v) The velocity at the moment of impact is

$$v(t_g) = -\sqrt{\frac{2|q|Eh}{m}}$$

or, taking its absolute value,

$$v_g = |v(t_g)| = \sqrt{\frac{2|q|Eh}{m}}.$$

(vi) Since $q < 0$ the potential energy can be rewritten to $E_{\text{pot}}(z) = |q|Ez$. This formula clearly is very similar to the gravitational potential energy (m, g and $|q|$, E are positive).
Now, energy conservation gives us

$$|q|Eh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v_g = \sqrt{\frac{2|q|Eh}{m}}.$$

(vii) The voltage between $z = h$ and the ground, $z = 0$, is indeed

$$U_h = \frac{-qEh - 0}{q} = -Eh.$$

Remark: So, the voltage is *negative* which is indeed correct. From the lecture you probably remember that voltage between two points is the difference of the *electric potentials* ϕ at the two points, here:

$$U_h = \phi(h) - \phi(0).$$

Now, $U_h < 0$ means $\phi(h) < \phi(0)$. This is the case just because the electric field points upwards (which probably wasn't the best choice of mine). If it would point downwards, then $E < 0$. Still, the equation $U_h = -Eh$ holds. But now $U_h > 0$.

Generally, you may remember: *The electric field alway points from a region with high electric potential to a region with low electric potential.*

It is important that you do not mix up electric potentials with potential energy: A *positive* charge has a high potential energy if it is in a region with *high* electric potential. But a *negative* charge (like the one in our exercise) has a high potential if it is in a region with a *low* electric potential.

Now, the formula $U = Ed$ (for the two plates) just doesn't care about signs. Here, $U > 0$ is the voltage between the positively charged plate (with high electric potential ϕ_1) and the negatively charged plate (with lower electric potential $\phi_2 = \phi_1 - U$).

3. Two parallel plates

Consider two large parallel square plates with edge length $l = 10\,\text{cm}$ with a distance $d = 1\,\text{cm}$. One of them is uniformly charged with $Q = 10^{-3}\,\text{C}$, the other one with $-Q$.

(a) Calculate the area A of each of the plates.

(b) What is the charge density σ?

(c) What is the electric field E between the plates?

(d) What is the voltage U between the plates?

(e) Imagine a point of charge $q = +10^{-8}\,\text{C}$ and mass $m = 0.05\,\text{kg}$ is put on the positively charged plate. It is repelled and moves towards the oppositely charged plate. With which velocity v does it reach the other plate?

4. Motion in homogeneous field

A point with charge q and mass m enters a region where an constant electric field E is present, which points "upward" in y-direction. Initially, the point moves in x-direction with velocity v_{x0}.

(a) Consider the force which acts on the point charge: What are its components F_x, F_y in x- and y-direction?

(b) Deduce the acceleration a_x, a_y (in both directions) which the point undergoes.

(c) What are the point's coordinates $x(t)$, $y(t)$ at time t, if initially (at $t = 0$) the particle is at $x(0) = y(0) = 0$?

(d) If the point has travelled a given distance l in y-direction, $y(t') = l$, what is its velocity $v_x(t')$, $v_y(t')$? How much kinetic energy has the point gained (as compared to the initial situation?).

5. Coulomb's Law

Consider a point with *positive* charge $q_1 = 10^{-6}\,\text{C}$. A second point with (unknown) charge q_2, located in a distance $d = 1\,\text{m}$, is *attracted* by the first one with a force $F_{12} = 20\,\text{N}$.

(a) Is the charge q_2 negative or positive?

(b) Calculate q_2.

(c) What would be the force F'_{12} if the distance were $d' = 2\,\text{m}$?

Notes: You may need $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Jm}}$.